Homework set 4

Note: the homework sets are not for submission. They are designed to help you prepare for the quizzes.

1. Consider the graph on Figure 1 (the number next to each edge specifies its length).

   (a) Write a linear program that computes the length of the shortest s-t path in this graph. The LP must be written explicitly, with both the objective function and all constraints given explicitly.

   (b) Convert the linear program into the canonical form.

   (c) Compute the dual of the linear program.

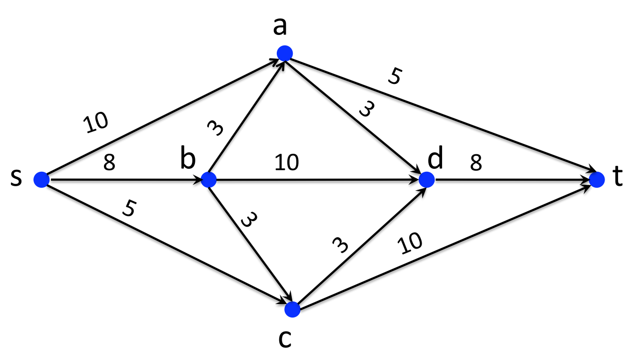

Figure 1: Graph with edge lengths

2. In this problem we consider the decision version of the shortest path problem: given a directed graph $G = (V, E)$ with integral non-negative edge lengths, two vertices s and t, and an integer k, the goal is to determine whether there is a path of length at most k connecting s to t in G. We denote this problem SP. For each one of the following two statements, decide whether it is true, false, or the answer depends on whether $P = NP$. Give a short explanation.

   (a) $SP \leq_p$ Set Cover.

   (b) $SAT \leq_p SP$.

3. A SAT formula φ is called *monotone*, iff each clause in φ only contains non-negated variables. For example $(x_1 \vee x_3 \vee x_2) \wedge (x_1 \vee x_5 \vee x_7) \wedge (x_1 \vee x_4 \vee x_6)$ is a monotone formula, while $(x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee x_4)$ is not. Notice that a monotone SAT formula always has a satisfying assignment - the one in which the values of all variables are set to TRUE. In the Restricted Monotone Satisfiability problem, we are given a monotone SAT formula φ and an integer k, and we need to establish whether there is a satisfying assignment to φ in which exactly k of the variables are assigned the value TRUE, and the rest of the variables are set to FALSE. Prove that Restricted Monotone Satisfiability is NP-complete.

4. We are given a collection of n items, whose weights are $w_1, w_2, \ldots, w_n$. We need to move these items from one city to another in trucks. The maximum amount of weight that each truck is allowed to carry is K. We would like to move all these items using the minimum possible number of trucks.

   Consider the following simple algorithm. Start loading the items one-by-one into the first truck, until we reach an item whose addition would overflow the weight limit on that truck. We then declare the first truck full and send it off. We then continue to the second truck with the remaining items, and so on.

   (a) Prove that this algorithm does not always produce an optimal solution to the problem.

   (b) Prove that the algorithm always produces a factor-2 approximation.