Algorithms Tutorial 3

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Question 0.1. *You are given a rooted tree T. The root contains a message m which has to be communicated to all nodes in T. Communicating m from u to v ($uv \in T$) takes one timestep. Furthermore each node can only communicate with one of its neighbours in a given timestep. Note that in one timestep many nodes may transfer m to a corresponding neighbour in parallel. Design an algorithm to determine the the minimum number of timesteps $C(T)$ to distribute the message throughout T.*

This is problem 16 on pg. 327 of the Kleinberg & Tardos book.

Here is a sketch of the solution.
Suppose r is the root of T. Further suppose that we know the optimal way to schedule the communications for each subtree T_i for $i \in N(r)$ (the neighbours of r). A natural strategy is to simply communicate m to each $i \in N(r)$ from r in decreasing order of communication time $C(T_i)$ (ties broken arbitrarily). This suggests a natural linear time bottom-up algorithm.

To see that the strategy is optimal we simply use induction on the depth d of T. The base case $d = 0$ is trivial. Suppose the hypothesis holds for trees of depth atmost $d-1$. Now one can show (say by contradiction) that T with depth d has an optimal communication strategy given by the algorithm above.