Let A be a fixed set of equations and $\prec$ a fixed term rewriting order.

Let H, J, and E range over sets of equations.

Definition: $A \models_{ind} a = b$ if for every ground instance $a' = b'$ of $a = b$ (over a given fixed set of constant and funciton symbols) we have $A \models a' = b'$.

Example:

$$0 + x = x, \;\; s(x) + y = s(x + y) \not\models x + y = y + x$$

but

$$0 + x = x, \;\; s(x) + y = s(x + y) \models_{ind} x + y = y + x$$

assuming that 0, s, and $+$ are all the constants and functions.

The problem is to give inference rules for deriving sequents of the form $A \models_{ind} a = b$.

Definition: $H \models_{\prec,A} E$ means that for any ground instance $a = b$ of an equation in E, if for all ground instances $c = d$ of an equation in H such that $c = d \prec a = b$ (using the multiset extension of $\prec$ to two element multisets) we have that $A \models c = d$, then we have $A \models a = b$.

Intuitively, $H \models_{\prec,A} E$ means that each equation in E is provable given H as induction hypotheses.

Definition: I will write $b \rightarrow_{\prec,J} b'$ to mean that b can be written as $C[w]$ where b' is $C[w']$ and $w = w'$ is an instance of an equation in J such that $w' \prec w$.

Definition: I will write $a = b \rightarrow_{\prec,A,H} a' = b'$ if $a' = b'$ is different from $a = b$, $a \rightarrow^*_{\prec,A \cup H} a'$, $b \rightarrow^*_{\prec,A \cup H} b'$, and every instance of H used in the rewriting is less than $a = b$ under the multiset extension of $\prec$.

Definition: I will use the notation $\{x_i\}_i$ to denote a set whose elements x_i are indexed by i. A set of substitutions $\{\sigma_i\}_i$ is called a *cover set* if for every ground substitution (over a given fixed set of constant and function symbols) γ there exists some σ_i in $\{\sigma_i\}_i$ and some substitution γ' such that for any variable x we have $\gamma(x) \rightarrow^*_{\prec,A} \gamma'(\sigma_i(x))$.

Sound Inference Rules

Reddy's forward chaining rules:

I^R

Start	$H \models_{\prec,A} \emptyset$	
Expand	$\frac{H \cup \{a=b\} \models_{\prec,A} E}{H \cup H' \models_{\prec,A} E \cup \{a=b\}}$	if $a \succ b$ $\{\sigma_i\}_i$ is a cover set of substitutions and $H' = \{c = b\sigma_i \mid a\sigma_i \rightarrow_{\prec,A} c\}_i$
Simplify	$\frac{H \cup \{a=b\} \models_{\prec,A} E}{H \cup \{a'=b\} \models_{\prec,A} E}$	if $a \rightarrow_{\prec,A \cup E} a'$
Reduce	$\frac{H \cup \{a=a\} \models_{\prec,A} E}{H \models_{\prec,A} E}$	
Finish	$\frac{\emptyset \models_{\prec,A} E}{A \models_{ind} E}$	

McAllester's Backward Chaining Rules:

I^M

Start	$\frac{H \models_{\prec,A} H}{A \models_{ind} H}$	
Expand	$\frac{H \models_{\prec,A} E \cup E'}{H \models_{\prec,A} E \cup \{a=b\}}$	if $\{\sigma_i\}_i$ is a cover set of substitutions and $E' = \{a\sigma_i = b\sigma_i\}_i$
Simplify	$\frac{H \cup \{a'=b'\} \models_{\prec,A} E \cup \{a'=b'\}}{H \models_{\prec,A} E \cup \{a=b\}}$	if $a = b \rightarrow_{\prec,A,H} a' = b'$
Reduce	$\frac{H \models_{\prec,A} E}{H \models_{\prec,A} E \cup \{a=a\}}$	
Finish	$H \models_{\prec,A} \emptyset$	