Visual Recognition: Transformations and Features

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TTI Chicago

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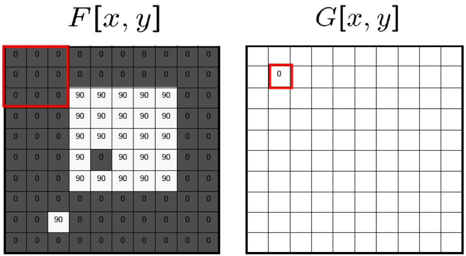
What did we see in class last week?

Last week

- Image formation.
- Filtering: convolution vs correlation



- Separable filters.
- Computing edges.
- Steerable filters.
- Midwest Vision Workshop.

First Recognition System: Template matching

- What if the template is not identical to some subimage in the scene?
- Match can be meaningful, if scale, orientation, and general appearance is right.
- How can I find the right scale?

Scene

Template

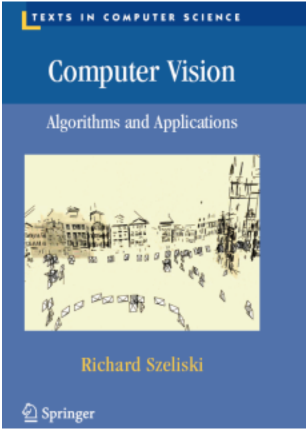
[Source: K. Grauman]

Today's lecture ...

- Additional transformations
- Local features: Interest point detection and descriptors

Material

- Chapter 3 and 4 of Rich Szeliski book



- Available online here

Other transformations

Integral Images

- If an image is going to be repeatedly convolved with different box filters, it is useful to compute the **summed area table**.
- It is the running sum of all the pixel values from the origin

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- To find the summed area (integral) inside a rectangle $[i_0, i_1] \times [j_0, j_1]$ we simply combine four samples from the summed area table.

$$S([i_0, i_1] \times [j_0, j_1]) = s(i_1, j_1) - s(i_1, j_0 - 1) - s(i_0 - 1, j_1) + s(i_0 - 1, j_0 - 1)$$

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Example of Integral Images

3	2	7	2	3
1	5	1	3	4
5	1	**3**	5	1
4	3	2	1	6
2	4	1	4	8

(a) S = 24

3	5	12	14	17
4	*11*	**19**	24	31
9	**17**	28	38	46
13	24	37	48	62
15	30	44	59	81

(b) s = 28

3	5	12	*14*	17
4	11	19	24	31
9	17	28	38	46
13	24	37	**48**	62
15	30	44	59	81

(c) S = 24

Figure 3.17 Summed area tables: (a) original image; (b) summed area table; (c) computation of area sum. Each value in the summed area table $s(i,j)$ (red) is computed recursively from its three adjacent (blue) neighbors (3.31). Area sums S (green) are computed by combining the four values at the rectangle corners (purple) (3.32). Positive values are shown in **bold** and negative values in *italics*.

Non-linear filters: Median filter

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Example of non-linear filters

1	2	1	2	4
2	1	3	5	8
1	3	7	6	9
3	4	8	6	7
4	5	7	8	9

(Median filter)

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(α-trimmed mean)

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- Weighted filter kernel with a better outlier rejection.
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- Data-dependent bilateral weight function

$$w(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2} - \frac{||f(i,j) - f(k,l)||^2}{2\sigma_r^2}\right)$$

composed of the **domain kernel** and the **range kernel**.

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Example Bilateral Filtering

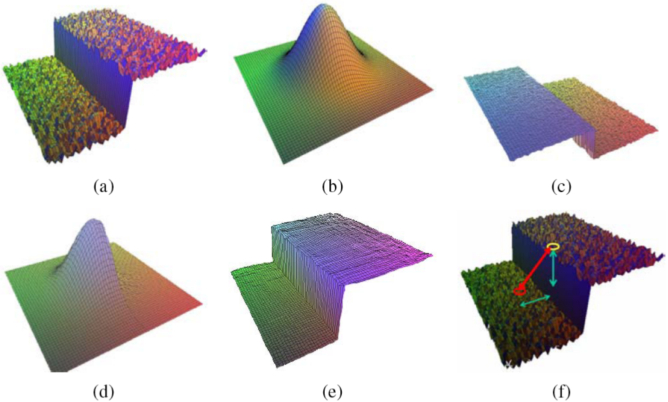

Figure: Bilateral filtering [Durand & Dorsey, 02]. (a) noisy step edge input. (b) domain filter (Gaussian). (c) range filter (similarity to center pixel value). (d) bilateral filter. (e) filtered step edge output. (f) 3D distance between pixels

[Source: R. Szeliski]

Distance Transform

- Useful to quickly precomputing the distance to a curve or a set of points.
- Let $d(k, l)$ be some distance metric between pixel offsets, e.g., Manhattan distance
$$d(k, l) = |k| + |l|$$
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- The distance transform $D(i, j)$ of a binary image $b(i, j)$ is defined as

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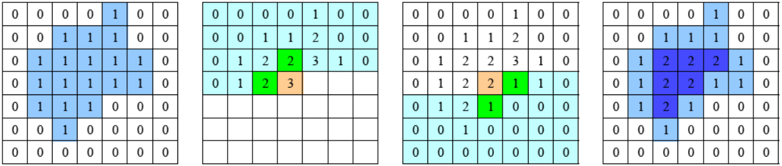

(a) (b) (c) (d)

Figure: City block distance transform: (a) original binary image; (b) top to bottom (forward) raster sweep: green values are used to compute the orange value; (c) bottom to top (backward) raster sweep: green values are merged with old orange value; (d) final distance transform.

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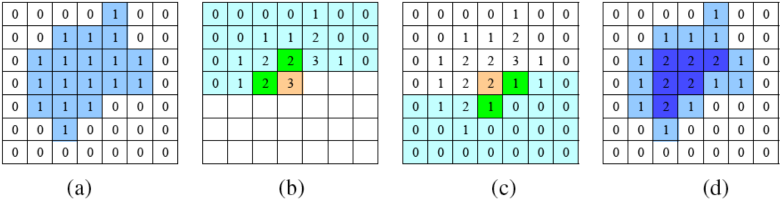

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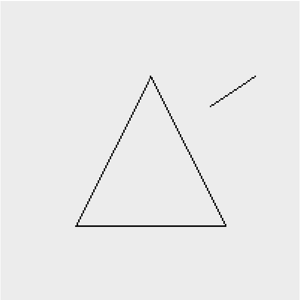
[Source: R. Szeliski]

Example of Distance Transform

- More complicated in the Euclidean case.
- Example of a distance transform



- The ridges is the **skeleton** or **medial axis**.
- Extension: Signed distance transform.

[Source: P. Felzenszwalb]

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- Pass a sinusoid of known frequency through the filter and to observe by how much it is attenuated

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- If we convolve the sinusoidal signal $s(x)$ with a filter whose impulse response is $h(x)$, we get another sinusoid of the same frequency but different magnitude and phase

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Filtering and Fourier

- Convolution can be expressed as a weighted summation of shifted input signals (sinusoids); so it is just a single sinusoid at that frequency.

$$o(x) = h(x) * s(x) = A \sin(\omega x + \phi_o)$$

A is the **gain** or **magnitude** of the filter, while the phase difference $\Delta\phi = \phi_o - \phi_i$i is the **shift** or **phase**

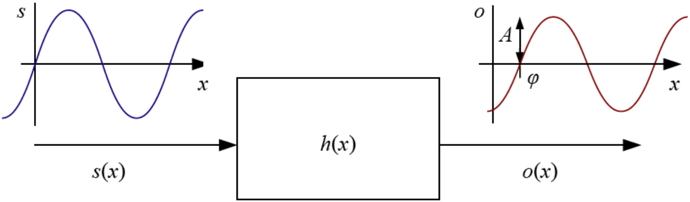

Figure 3.24 The Fourier Transform as the response of a filter $h(x)$ to an input sinusoid $s(x) = e^{j\omega x}$ yielding an output sinusoid $o(x) = h(x) * s(x) = Ae^{j\omega x+\phi}$.

Complex notation

- The sinusoid is express as $s(x) = e^{j\omega x} = \cos \omega x + j \sin \omega x$ and the filter sinusoid as

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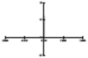
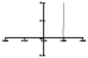
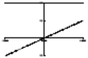
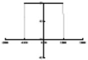
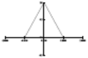
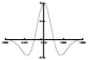
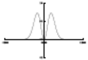
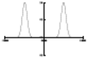
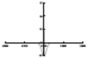
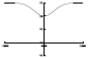
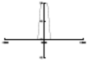
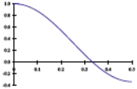
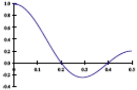
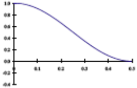
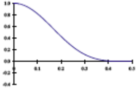
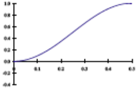
Properties Fourier Transform

Property	Signal		Transform
superposition	$f_1(x) + f_2(x)$		$F_1(\omega) + F_2(\omega)$
shift	$f(x - x_0)$		$F(\omega)e^{-j\omega x_0}$
reversal	$f(-x)$		$F^*(\omega)$
convolution	$f(x) * h(x)$		$F(\omega)H(\omega)$
correlation	$f(x) \otimes h(x)$		$F(\omega)H^*(\omega)$
multiplication	$f(x)h(x)$		$F(\omega) * H(\omega)$
differentiation	$f'(x)$		$j\omega F(\omega)$
domain scaling	$f(ax)$		$1/aF(\omega/a)$
real images	$f(x) = f^*(x)$	$\Leftrightarrow$	$F(\omega) = F(-\omega)$
Parseval's Theorem	$\sum_x [f(x)]^2$	$=$	$\sum_\omega [F(\omega)]^2$

[Source: R. Szeliski]

Name	Signal			Transform	
impulse		$\delta(x)$	$\Leftrightarrow$	1	
shifted impulse		$\delta(x-u)$	$\Leftrightarrow$	$e^{-j\omega u}$	
box filter		$\mathrm{box}(x/a)$	$\Leftrightarrow$	$a\,\mathrm{sinc}(a\omega)$	
tent		$\mathrm{tent}(x/a)$	$\Leftrightarrow$	$a\,\mathrm{sinc}^2(a\omega)$	
Gaussian		$G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega;\sigma^{-1})$	
Laplacian of Gaussian		$(\frac{x^2}{\sigma^4}-\frac{1}{\sigma^2})G(x;\sigma)$	$\Leftrightarrow$	$-\frac{\sqrt{2\pi}}{\sigma}\omega^2 G(\omega;\sigma^{-1})$	
Gabor		$\cos(\omega_0 x)G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega \pm \omega_0;\sigma^{-1})$	
unsharp mask		$(1+\gamma)\delta(x)$ $-\gamma G(x;\sigma)$	$\Leftrightarrow$	$(1+\gamma)-$ $\frac{\sqrt{2\pi}\gamma}{\sigma}G(\omega;\sigma^{-1})$	
windowed sinc		$\mathrm{rcos}(x/(aW))$ $\mathrm{sinc}(x/a)$	$\Leftrightarrow$	(see Figure 3.29)	

[Source: R. Szeliski]

Name	Kernel	Transform	Plot
box-3	$\frac{1}{3}\boxed{1}\boxed{1}\boxed{1}$	$\frac{1}{3}(1 + 2\cos\omega)$	
box-5	$\frac{1}{5}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}$	$\frac{1}{5}(1 + 2\cos\omega + 2\cos 2\omega)$	
linear	$\frac{1}{4}\boxed{1}\boxed{2}\boxed{1}$	$\frac{1}{2}(1 + \cos\omega)$	
binomial	$\frac{1}{16}\boxed{1}\boxed{4}\boxed{6}\boxed{4}\boxed{1}$	$\frac{1}{4}(1 + \cos\omega)^2$	
Sobel	$\frac{1}{2}\boxed{-1}\boxed{0}\boxed{1}$	$\sin\omega$	
corner	$\frac{1}{2}\boxed{-1}\boxed{2}\boxed{-1}$	$\frac{1}{2}(1 - \cos\omega)$	

[Source: R. Szeliski]

2D Fourier Transform

- Same as 1D, but in 2D. Now the sinusoid is

$$s(x, y) = \sin(\omega_x x + \omega_y y)$$

- The 2D Fourier in continuous domain is then

$$H(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) e^{-j\omega_x x + \omega_y y} dxdy$$

and in the discrete domain

$$H(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} h(x, y) e^{-2\pi j \frac{k_x x + k_y y}{MN}}$$

where M and N are the width and height of the image.

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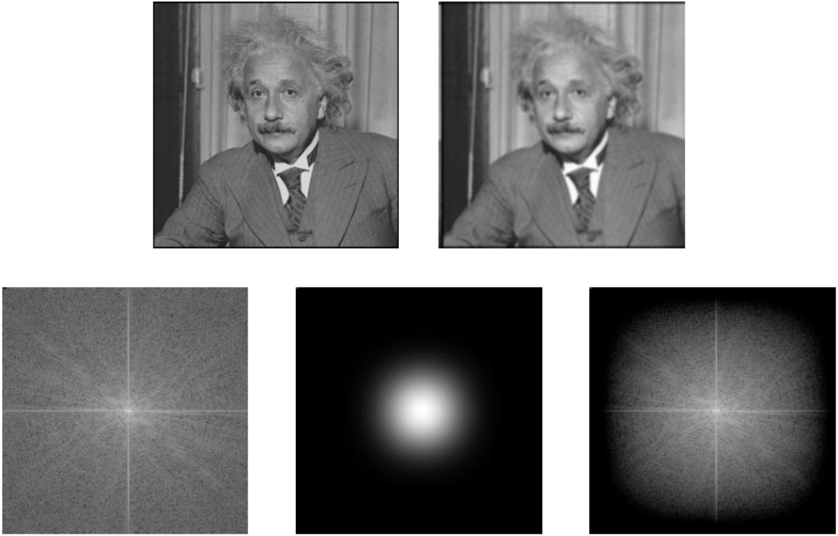
Example of 2D Fourier Transform



[Source: A. Jepson]

Pyramids

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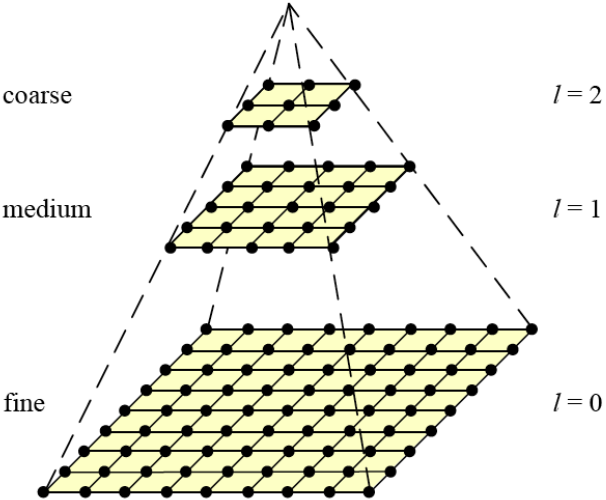
Image Pyramid



[Source: R. Szeliski]

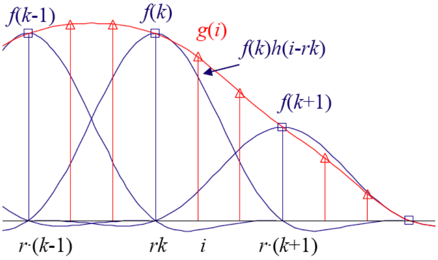
Interpolation

- To **interpolate** (or upsample) an image to a higher resolution, we need to select an interpolation kernel with which to convolve the image

$$g(i,j) = \sum_{k,l} f(k,l)h(i - rk, j - rl)$$

with r the up-sampling rate.

- The linear interpolator (corresponding to the tent kernel) produces interpolating piecewise linear curves.
- More complex kernels, e.g., B-splines.



[Source: R. Szeliski]

Decimation

- **Decimation**: reduces resolution

$$g(i,j) = \sum_{k,l} f(k,l)h(i - k/r, j - l/r)$$

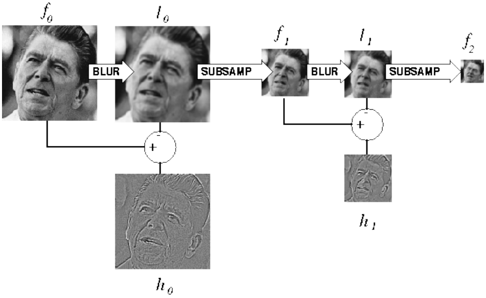
  with r the down-sampling rate.

- Different filters exist as well.

Multi-Resolution Representations

The most used one is the Laplacian pyramid:

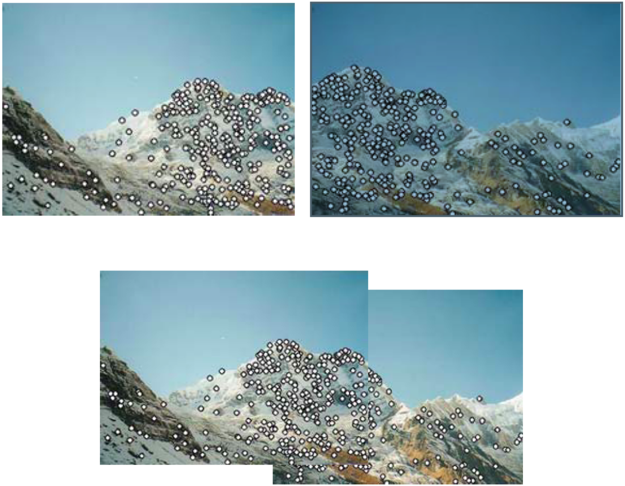
- We first blur and subsample the original image by a factor of two and store this in the next level of the pyramid.
- They then subtract this low-pass version from the original to yield the band-pass Laplacian image.
- The pyramid has perfect reconstruction: the Laplacian images plus the base-level Gaussian are sufficient to exactly reconstruct the original image.



- How do we reconstruct back?

Local features for **instance-level** recognition

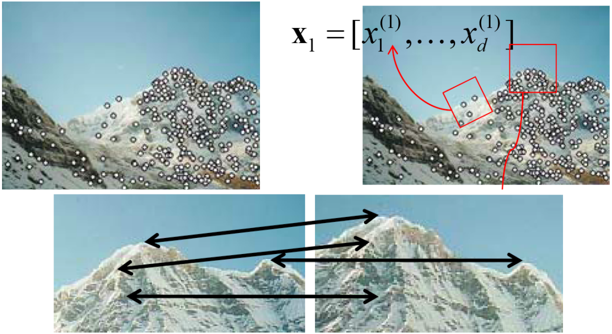
Application Example: Image stitching



[Source: K. Grauman]

Local features

- **Detection**: Identify the interest points.
- **Description**: Extract vector feature descriptor around each interest point.
- **Matching**: Determine correspondence between descriptors in two views.
- **Tracking**: alternative to matching that only searches a small neighborhood around each detected feature.



[Source: K. Grauman]

Goal: interest operator repeatability

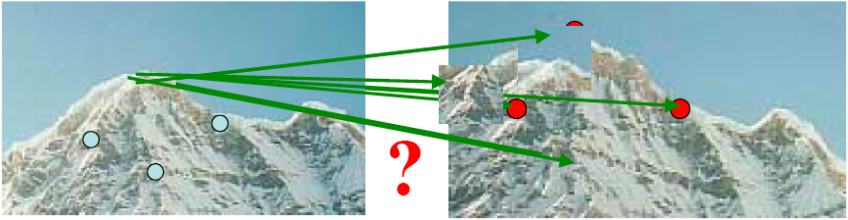
- We want to detect (at least some of) the same points in both images.
- We have to be able to run the detection procedure independently per image.

Figure: No chance to find the true matches

[Source: K. Grauman]

Goal: descriptor distinctiveness

- We want to be able to reliably determine which point goes with which.
- Must provide some invariance to geometric and photometric differences between the two views.



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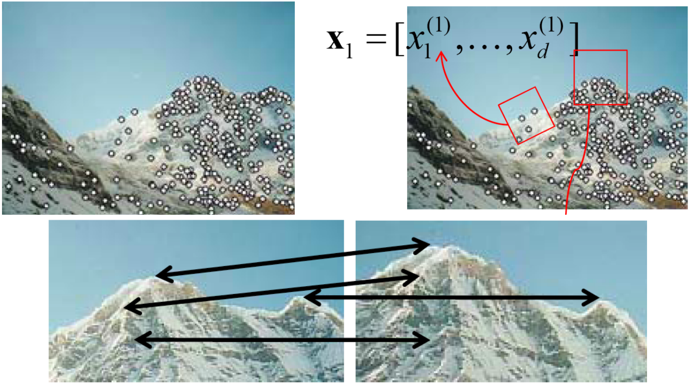

[Source: K. Grauman]

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Corners as distinctive interest points

- We should easily recognize the point by looking through a small window.
- Shifting a window in any direction should give a large change in intensity.

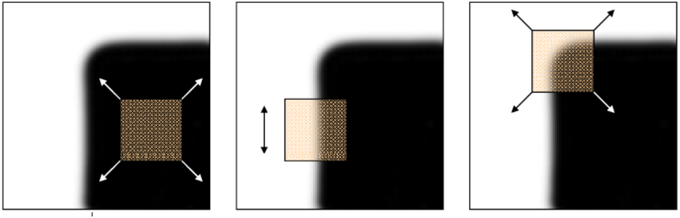

Figure: (left) flat region: no change in all directions, (center) edge: no change along the edge direction, (right) corner: significant change in all directions

[Source: Alyosha Efros, Darya Frolova, Denis Simakov]

A Simple Matching Criteria

- Compare two image patches using (weighted) summed square difference

$$E_{WSSD}(\mathbf{u}) = \sum_i w(\mathbf{p}_i)[I_1(\mathbf{p}_i + \mathbf{u}) - I_0(\mathbf{p}_i)]^2$$

  with I_0 and I_1 two images being compared, $\mathbf{u}(u_x, u_y)$ a displacement vector, $w(\mathbf{p})$ a spatially varying weighting function, and the summation i is over all the pixels in the patch.

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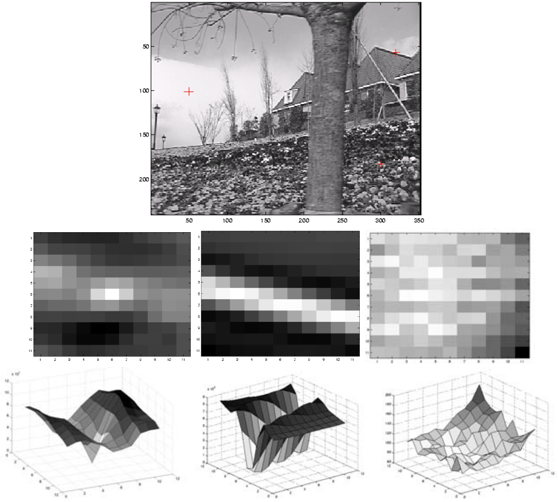
Which one is better?



[Source: R. Szeliski]

How to select?

- Using a Taylor Series expansion $I_0(\mathbf{p}_i + \Delta\mathbf{u}) \approx I_0(\mathbf{p}_i) + \nabla I_0(\mathbf{p}_i)$ we can approximate the autocorrelation as

$$
\begin{aligned}
E_{AC}(\Delta\mathbf{u}) &= \sum_i w(\mathbf{p}_i)[I_0(\mathbf{p}_i + \Delta\mathbf{u}) - I_0(\mathbf{p}_i)]^2 \\
&\approx \sum_i w(\mathbf{p}_i)[I_0(\mathbf{p}_i) + \nabla I_0(\mathbf{p}_i)\Delta\mathbf{u} - I_0(\mathbf{p}_i)]^2 \\
&= \sum_i w(\mathbf{p}_i)[\nabla I_0(\mathbf{p}_i)\Delta\mathbf{u}]^2 \\
&= \Delta\mathbf{u}^T \mathbf{A} \Delta\mathbf{u}
\end{aligned}
$$

  with

$$
\nabla I_0(\mathbf{p}_i) = \left(\frac{\partial I_0}{\partial x}, \frac{\partial I_0}{\partial y}\right)(\mathbf{p}_i)
$$

  the image gradient.

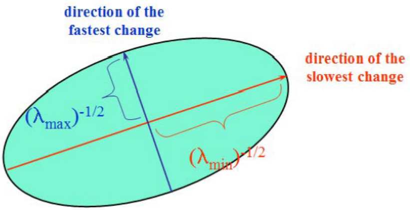
- Gradient can be computed with the filtering techniques we saw, e.g., derivatives of Gaussians.

More on selection

- The autocorrelation is $E_{AC}(\Delta\mathbf{u}) = \Delta\mathbf{u}^T \mathbf{A} \Delta\mathbf{u}$, with

$$\mathbf{A} = \sum_u \sum_v w(u,v) \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix} = w * \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}$$

  where we have replaced the weighted summations with discrete convolutions with the weighting kernel w.
- **A** can be interpreted as a tensor where the outer products of the gradients are convolved with a weighting function.
- Eigenvalues a notion of uncertainty



[Source: R. Szeliski]

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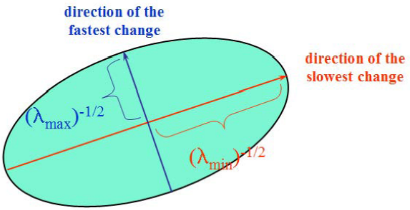


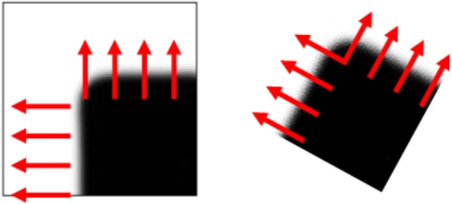
[Source: R. Szeliski]

Eigenvalues a notion of uncertainty

- $\mathbf{A}$ is symmetric

$$\mathbf{A} = \mathbf{U} \begin{bmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{bmatrix} \mathbf{U}^T \quad \text{with} \quad \mathbf{A}\mathbf{u}_i = \lambda_i \mathbf{u}_i$$

- The eigenvalues of $\mathbf{A}$ reveal the amount of intensity change in the two principal orthogonal gradient directions in the window.
- How is this matrix for



[Source: R. Szeliski]

Local Feature Selection Criteria

- Shi and Tomasi, 94 proposed the smallest eigenvalue of **A**, i.e., $\lambda_0^{-1/2}$.
- Harris and Stephens, 88 is rotationally invariant and downweights edge-like features where $\lambda_1 \gg \lambda_0$

$$\det(\mathbf{A}) - \alpha \mathrm{trace}(\mathbf{A})^2 = \lambda_0 \lambda_1 - \alpha(\lambda_0 + \lambda_1)^2$$

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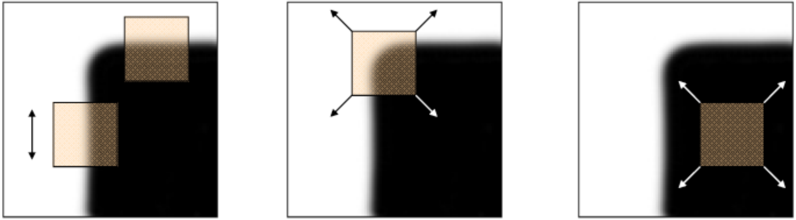
Type of responses



"edge":
$\lambda_1 >> \lambda_2$
$\lambda_2 >> \lambda_1$

"corner":
λ_1 and λ_2 are large,
$\lambda_1 \sim \lambda_2$;

"flat" region
λ_1 and λ_2 are small;

[Source: K. Grauman]

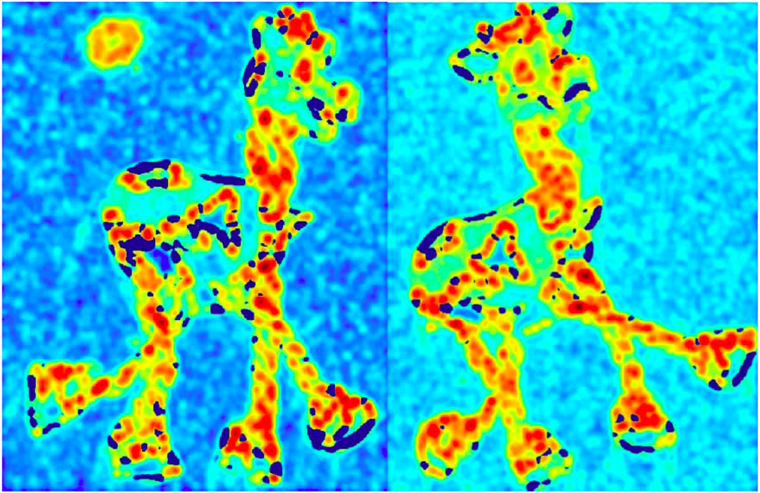
Harris Corner detector

1. Compute **A** for each image window to get their cornerness scores.
2. Find points whose surrounding window gave large corner response ($f >$ threshold).
3. Take the points of local maxima, i.e., perform non-maximum suppression.

Example

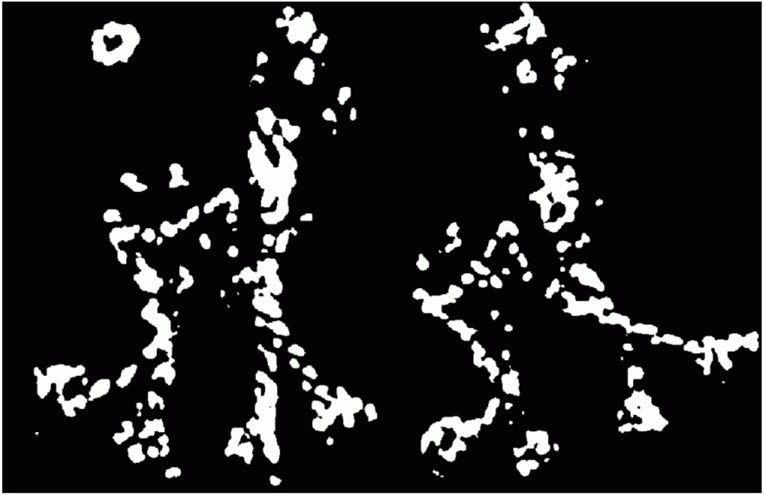
[Source: K. Grauman]

1) Compute Cornerness



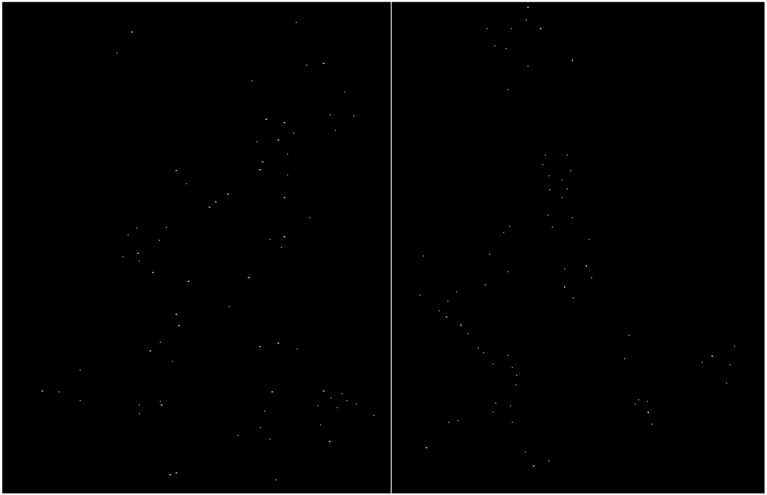
[Source: K. Grauman]

2) Find High Response



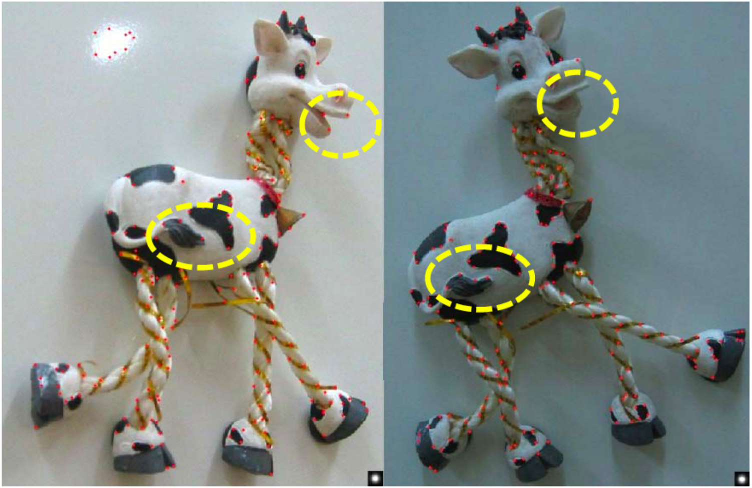
[Source: K. Grauman]

3) Non-maxima Suppresion



[Source: K. Grauman]

Results

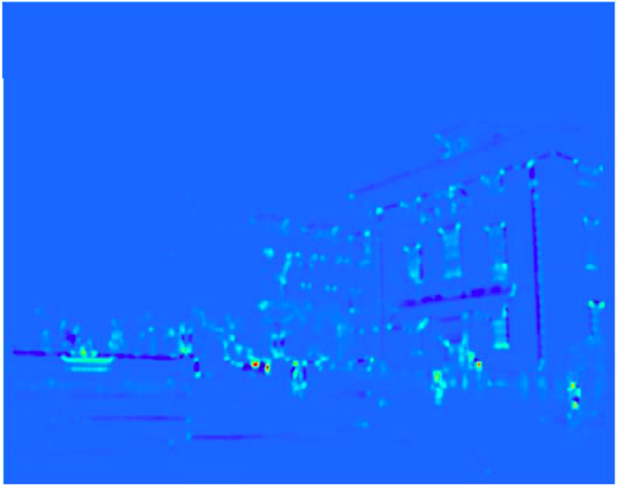


[Source: K. Grauman]

Another Example

[Source: K. Grauman]

Cornerness



[Source: K. Grauman]

Interest Points

[Source: K. Grauman]

Properties of Harris Corner Detector

- Rotation invariant?

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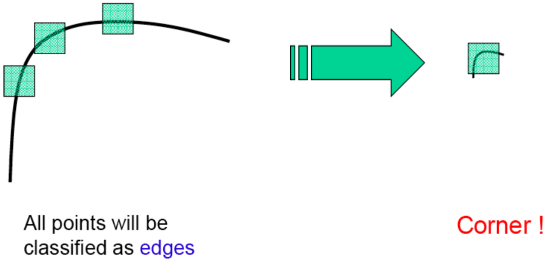
- Scale Invariant?



[Source: K. Grauman]

Scale invariant interest points

How can we independently select interest points in each image, such that the detections are repeatable across different scales?

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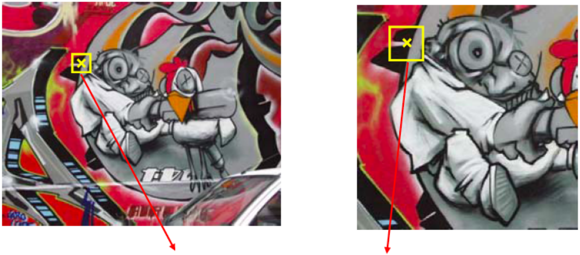
- More efficient to extract features that are stable in both location and scale.
- Find scale that gives local maxima of a function f in both position and scale.

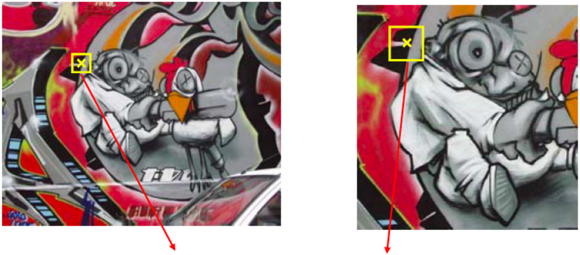


$$f(I_{i_1 \ldots i_m}(x, \sigma)) = f(I_{i_1 \ldots i_m}(x', \sigma'))$$

Scale invariant interest points

How can we independently select interest points in each image, such that the detections are repeatable across different scales?

- Extract features at a variety of scales, e.g., by using multiple resolutions in a pyramid, and then matching features at the same level.
- When does this work?
- More efficient to extract features that are stable in both location and scale.
- Find scale that gives local maxima of a function f in both position and scale.



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Automatic Scale Selection

Function responses for increasing scale (scale signature).

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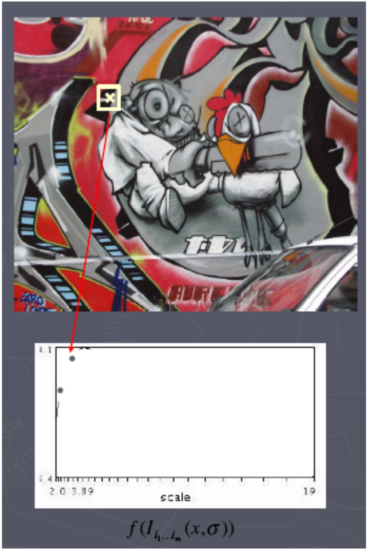

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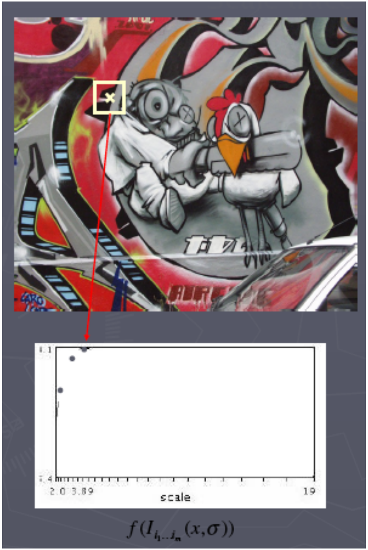

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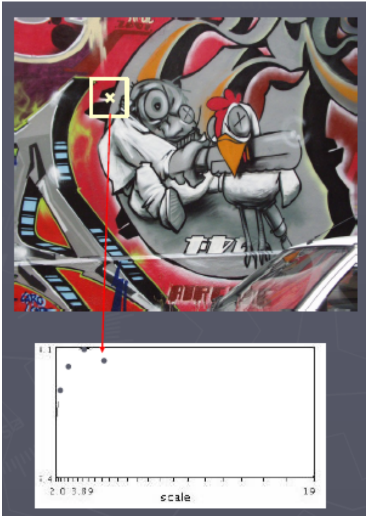

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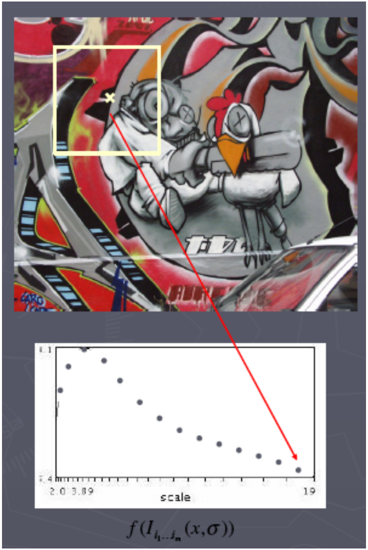

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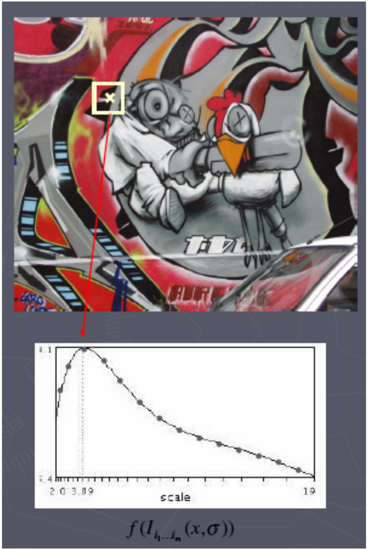

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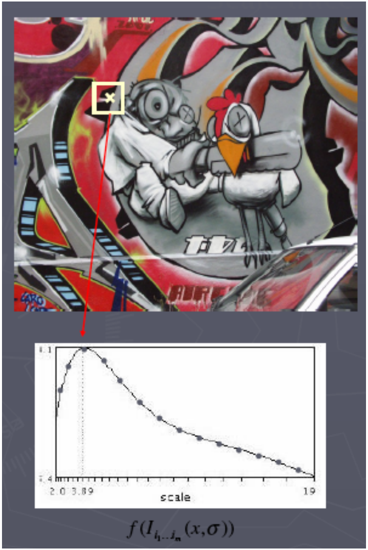

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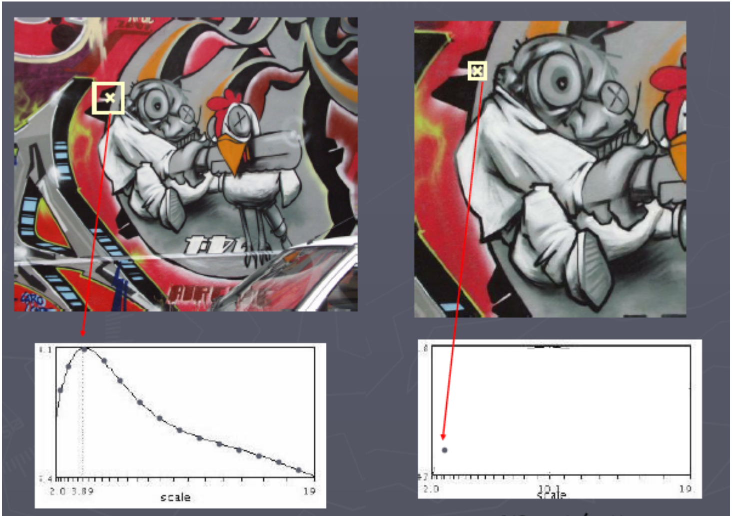

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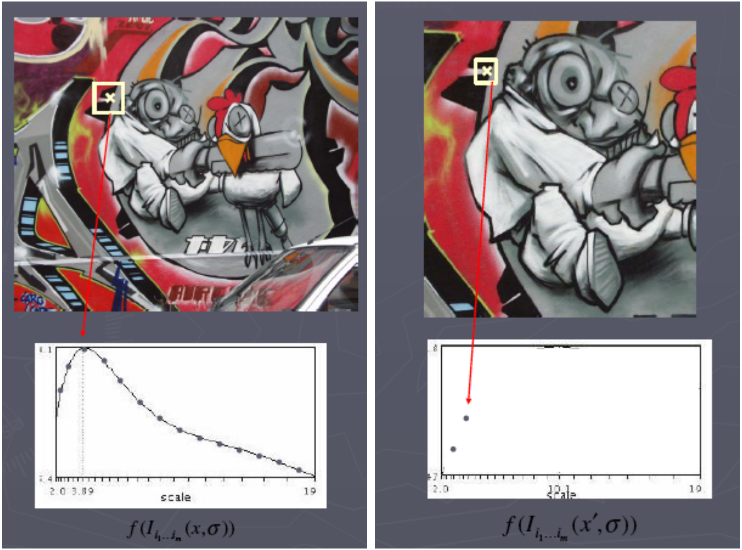

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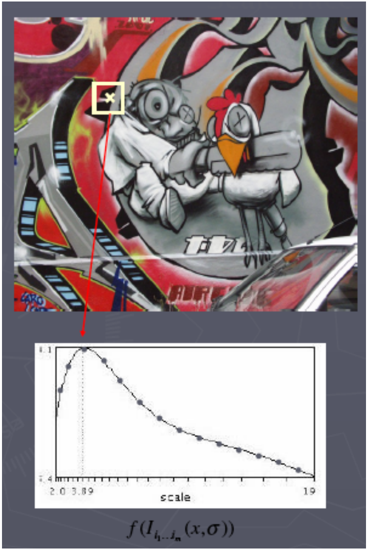

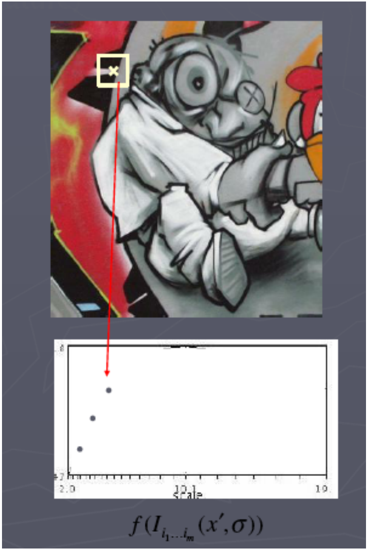

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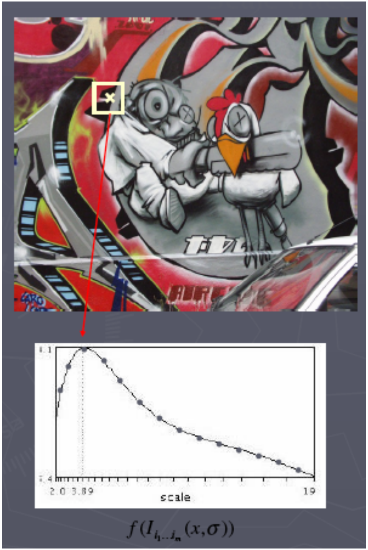

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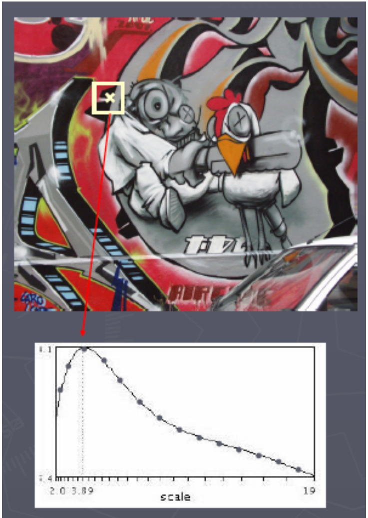

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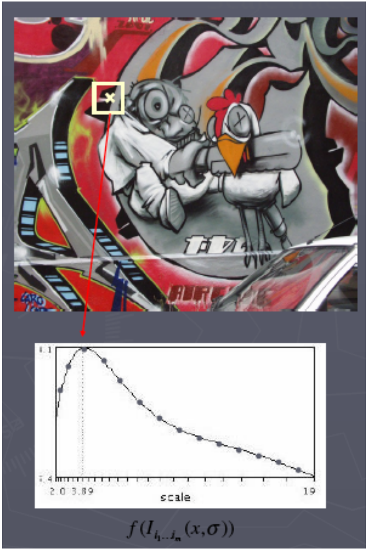
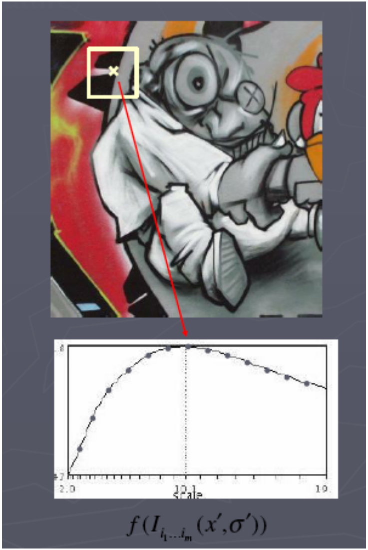
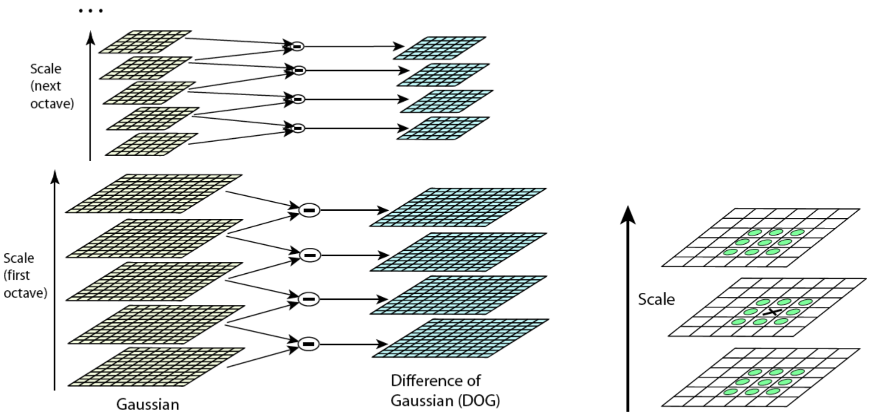




What can the signature function be?

- Lindeberg (1998): extrema in the Laplacian of Gaussian (LoG).
- Lowe (2004) proposed computing a set of sub-octave Difference of Gaussian filters looking for 3D (space+scale) maxima in the resulting structure.

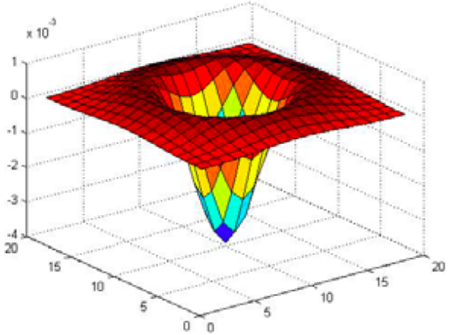


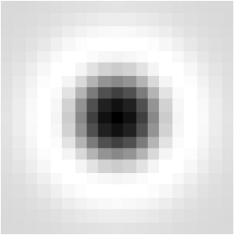
[Source: R. Szeliski]

Blob detection

- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

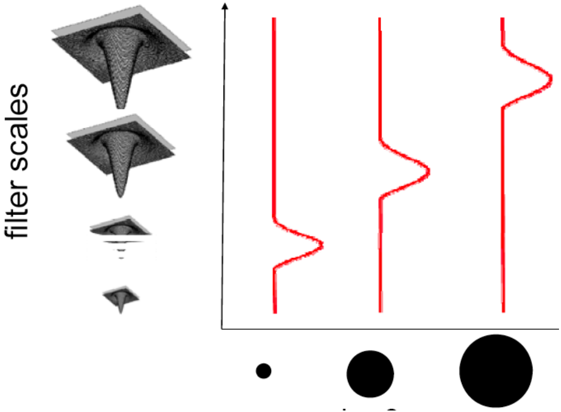




[Source: K. Grauman]

Blob detection in 2D: scale selection

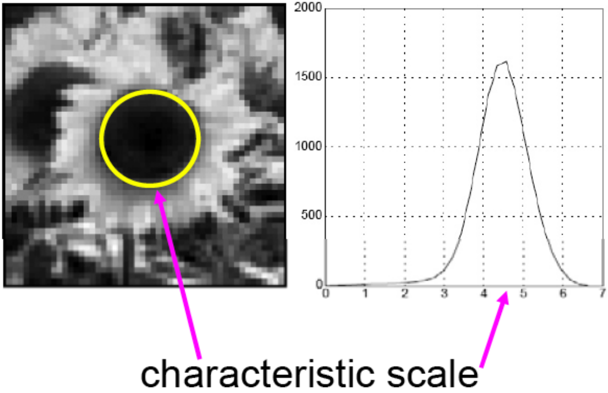
Laplacian-of-Gaussian = blob detector



[Source: B. Leibe]

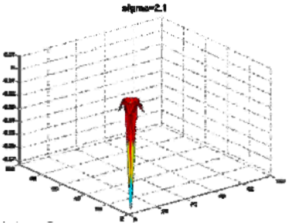
Characteristic Scale

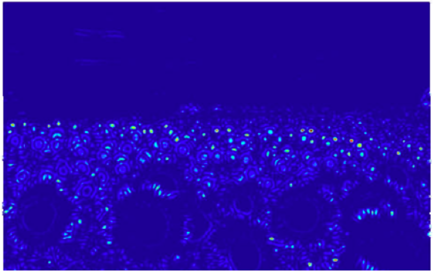
- We define the **characteristic scale** as the scale that produces peak of Laplacian response



[Source: S. Lazebnik]

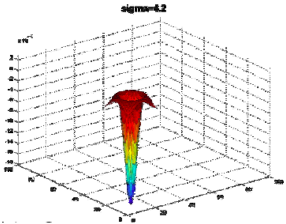
Example

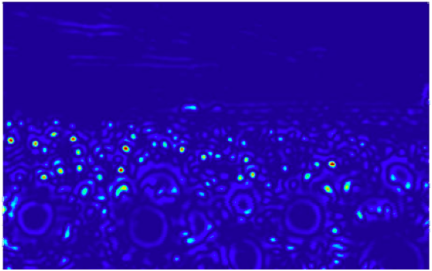




[Source: K. Grauman]

Example





[Source: K. Grauman]

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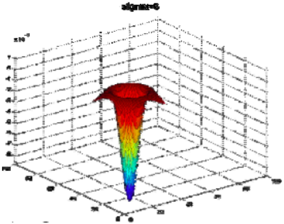
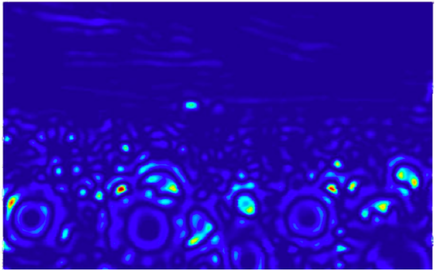
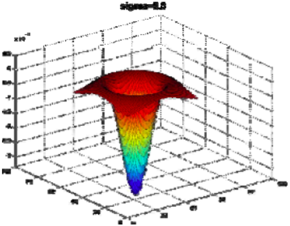
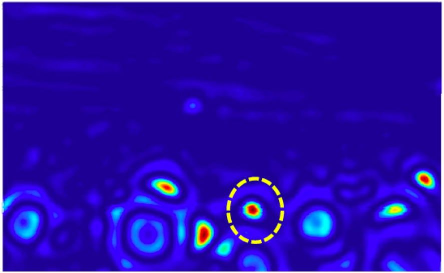




[Source: K. Grauman]

Example





[Source: K. Grauman]

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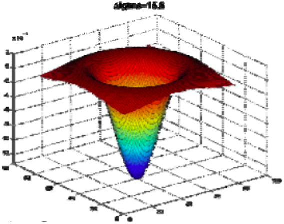

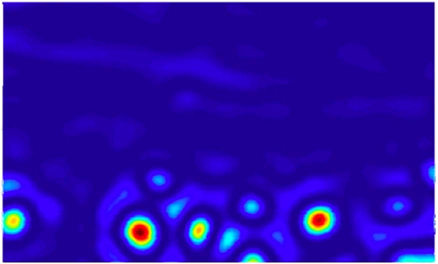

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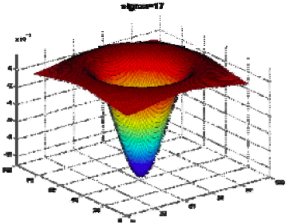
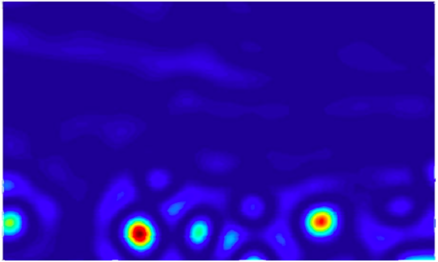
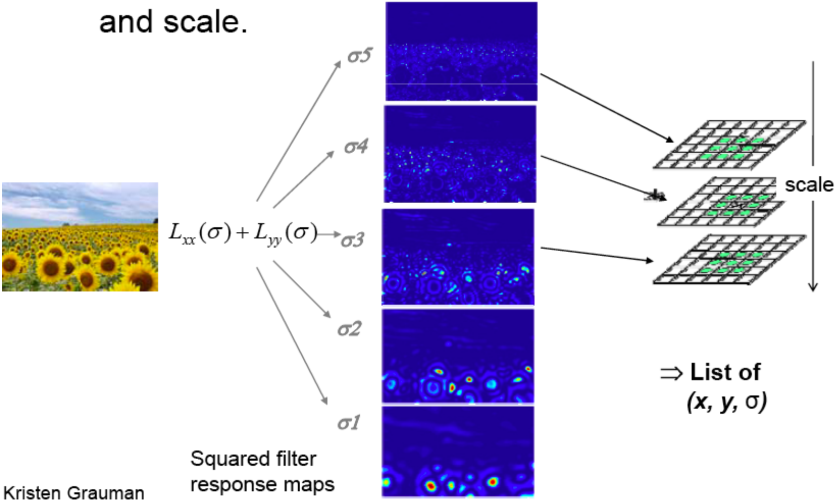




[Source: K. Grauman]

Scale invariant interest points

Interest points are local maxima in both position and scale.



Example

[Source: S. Lazebnik]

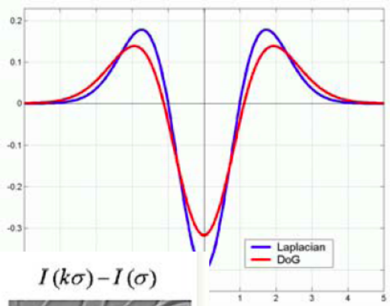
Fast approximation

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

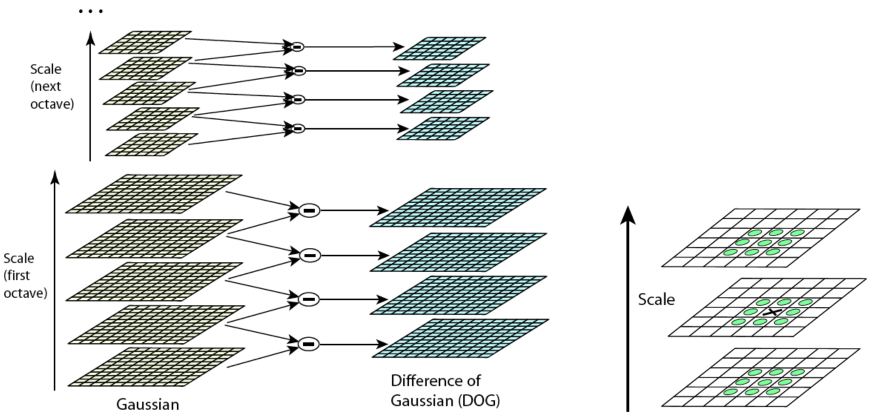
(Difference of Gaussians)



[Source: K. Grauman]

Lowe's DoG

- Lowe (2004) proposed computing a set of sub-octave Difference of Gaussian filters looking for 3D (space+scale) maxima in the resulting structure



[Source: R. Szeliski]

Laplacian vs Hessian

- Laplacian of Gaussians is scale invariant.
- Simple and efficient.
- But fires more on edges than determinant of hessian

Properties of the ideal feature

- **Local**: features are local, so robust to occlusion and clutter (no prior segmentation).
- **Invariant**: to certain transformations, e.g, scale, rotation.

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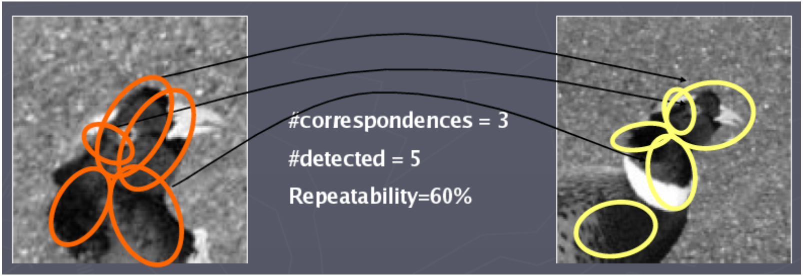
A lot of other interest point detectors

- Hessian
- Lowe: DoG
- Lindeberg: scale selection
- Miikolajczyk & Schmid: Hessian/Harris-Laplacian/Affine
- Tuyttelaars & Van Gool: EBR and IBR
- Matas: MSER
- Kadir & Brrady: Salient Regions
- Speeded–Up Robust Features (SURF) of Bay et al.

Evaluation criteria: repeatability

- Repeatability rate: percentage of detected that have correct corresponding points
- What's the problem of this?



[Source: T. Tuyttellaars]

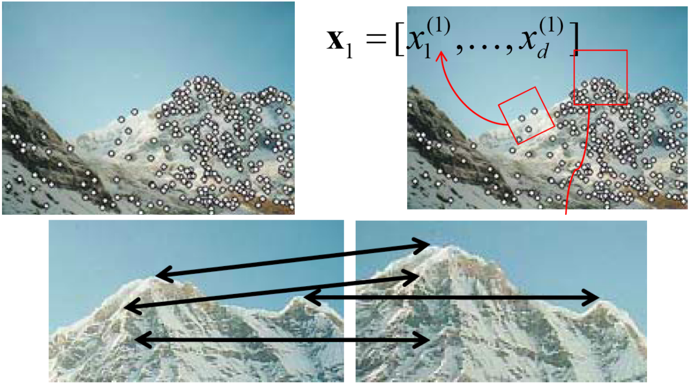
Evaluation criteria: repeatability

- Two points are in correspondence if the intersection over union is bigger than a certain threshold.

[Source: T. Tuyttellaars]

Local features

- **Detection**: Identify the interest points.
- **Description**: Extract vector feature descriptor around each interest point.
- **Matching**: Determine correspondence between descriptors in two views.



[Source: K. Grauman]

The ideal feature descriptor

- Repeatable (invariant/robust)
- Distinctive
- Compact
- Efficient

Invariances

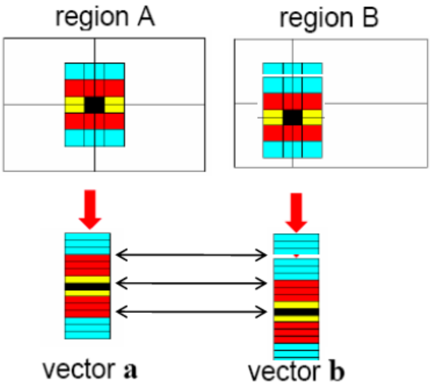
[Source: T. Tuytelaars]

Invariances

[Source: T. Tuytelaars]

Raw Pixels as Local Descriptrs

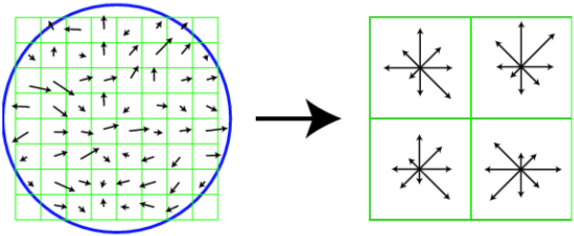
- The simplest way is to write down the list of intensities to form a feature vector, and normalize them (i.e., mean 0, variance 1).
- But this is very sensitive to even small shifts, rotations.



[Source: K. Grauman]

SIFT descriptor [Lowe 2004]

- Compute the gradient at each pixel in a 16×16 window around the detected keypoint, using the appropriate level of the Gaussian pyramid at which the keypoint was detected.
- Doweight gradients by a Gaussian fall-off function (blue circle) to reduce the influence of gradients far from the center.
- In each 4×4 quadrant, compute a gradient orientation histogram using 8 orientation histogram bins.



(a) image gradients (b) keypoint descriptor

[Source: R. Szeliski]

SIFT descriptor [Lowe 2004]

- To reduce the effects of location and dominant orientation misestimation, each of the original 256 weighted gradient magnitudes is softly added to $2 \times 2 \times 2$ histogram bins using trilinear interpolation.
- The resulting 128 non-negative values form a raw version of the SIFT descriptor vector.
- To reduce the effects of contrast or gain (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length.
- To further make the descriptor robust to other photometric variations, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.
- Great engineering effort!
- Why subpatches?
- Why does SIFT have some illumination invariance?

SIFT descriptor [Lowe 2004]

Extraordinarily robust matching technique

- Changes in viewpoint: up to about 60 degree out of plane rotation
- Changes in illumination: sometimes even day vs. night
- Fast and efficientcan run in real time
- Lots of code available

[Source: S. Seitz]

Example

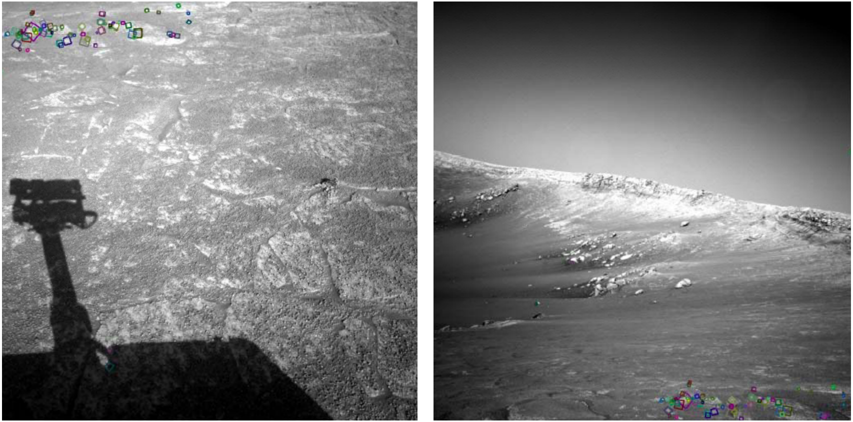


Figure: NASA Mars Rover images with SIFT feature matches

[Source: N. Snavely]

SIFT properties

Invariant to

- Scale
- Rotation

Partially invariant to

- Illumination changes
- Camera viewpoint
- Occlusion, clutter

Making descriptor rotation invariant

- Rotate patch according to its dominant gradient orientation
- This puts the patches into a canonical orientation

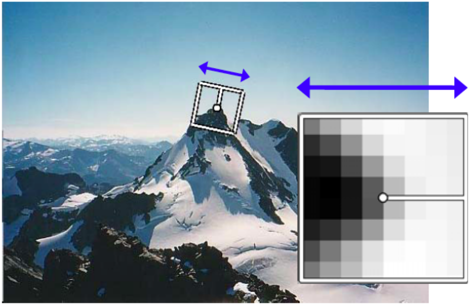
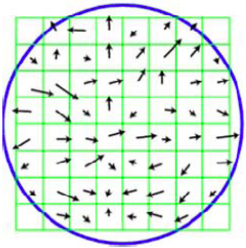


Figure: Figure from M. Brown

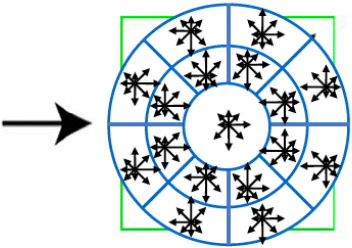
[Source: K. Grauman]

Gradient location-orientation histogram (GLOH)

- Developed by Mikolajczyk and Schmid (2005): variant on SIFT that uses a log-polar binning structure instead of the four quadrants.
- The spatial bins are 11, and 15, with eight angular bins (except for the central region), for a total of 17 spatial bins and 16 orientation bins.
- The 272D histogram is then projected onto a 128D descriptor using PCA trained on a large database.



(a) image gradients



(b) keypoint descriptor

[Source: R. Szeliski]

Other Descriptors

- Steerable filters
- moment invariants,
- complex filters
- shape contexts,,
- PCA-SIFT,
- HOG,
- SURF
- DAISY

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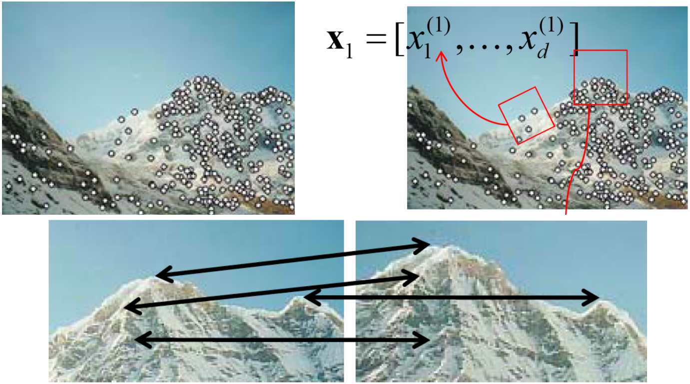


[Source: K. Grauman]

Matching local features

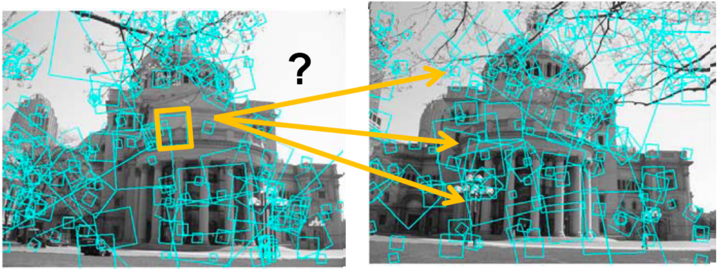
Once we have extracted features and their descriptors, we need to match the features between these images.

- Matching strategy: which correspondences are passed on to the next stage
- Devise efficient data structures and algorithms to perform this matching

Figure: Images from K. Grauman

Matching local features

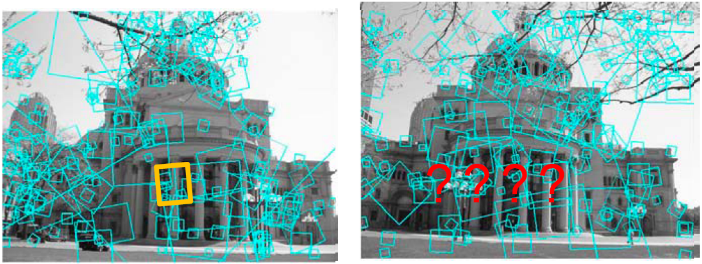
- To generate candidate matches, find patches that have the most similar appearance (e.g., lowest SSD)
- Simplest approach: compare them all, take the closest (or closest k, or within a thresholded distance)



[Source: K. Grauman]

Ambiguous matches

- At what SSD value do we have a good match?
- To add robustness, consider ratio of distance to best match to distance to second best match
  - If low, first match looks good.
  - If high, could be ambiguous match.



[Source: K. Grauman]

Matching SIFT Descriptors

- Nearest neighbor (Euclidean distance)
- Threshold ratio of nearest to 2nd nearest descriptor

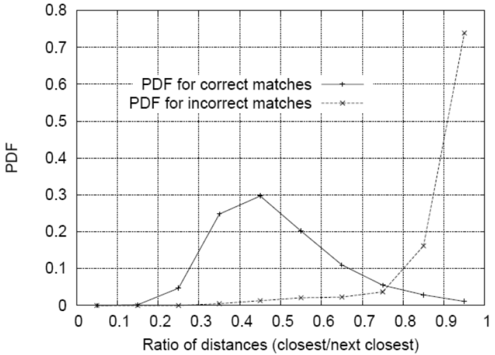


Figure: Images from D. Lowe

[Source: K. Grauman]