# Lecture 8
# Convolutional Neural Networks II

CMSC 35246: Deep Learning

Shubhendu Trivedi
&
Risi Kondor

University of Chicago

April 19, 2017

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    - Methods for Visualizing Convolutional Neural Networks

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    - Variations
    - Dilated Convolutions
    - Idea genealogy for Convolutional Neural Networks

# Housekeeping

- Quiz scores will be uploaded in a few hours
- Project proposals due tonight
- Mid term - 8 May (Just like quizzes, with some derivations)
- Late policy

# Convolutional Neural Networks

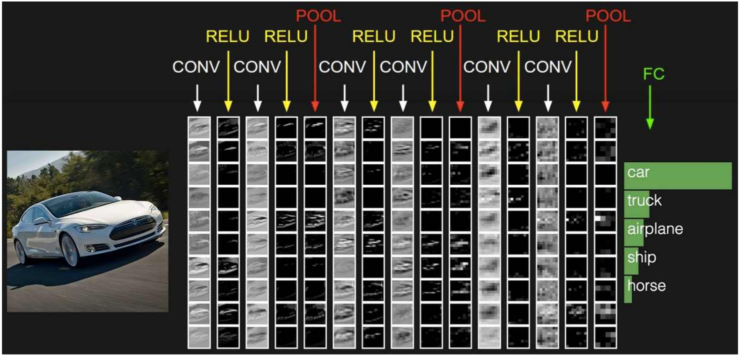


*Figure: Andrej Karpathy*

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- How do we probe what they actually learn, or do?

# A Global View: t-SNE

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- For nearby points $p_{j|i}$ will be high
- Suppose we had low dimensional maps $\mathbf{x}_i \mapsto \mathbf{y}_i$ and $\mathbf{x}_j \mapsto \mathbf{y}_j$

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- SNE aims to find a lower dimensional embedding such that the discrepancy between $p_{j|i}$ and $q_{j|i}$ is minimized
- Obvious cost function:

$$J = \sum_i KL(P_i||Q_i) = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$$

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- Modifies the cost function and uses a Student $t$-distribution to compute similarities in the low-dimensional space
- Takeaway: t-SNE embeds high dimensional points into a lower dimensional space so as to preserve pairwise distances
- In other words: Similar objects get embedded nearby

# t-SNE on MNIST

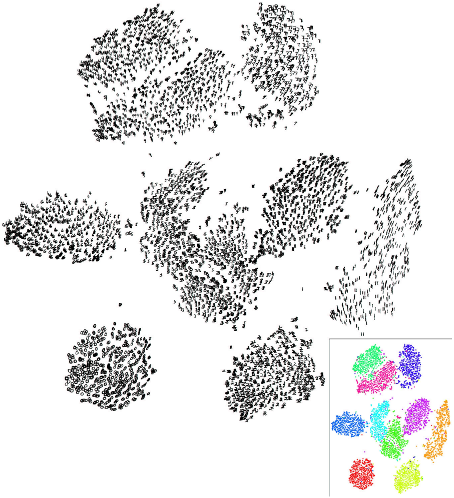
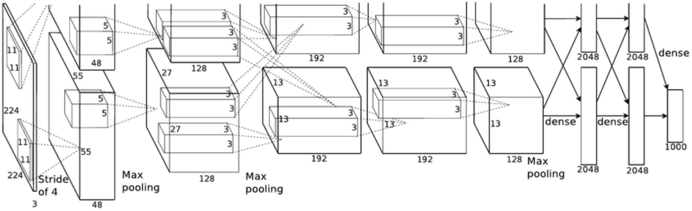


*Figure: van der Maaten and Hinton*

# AlexNet

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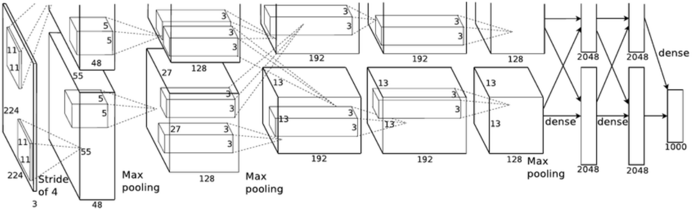
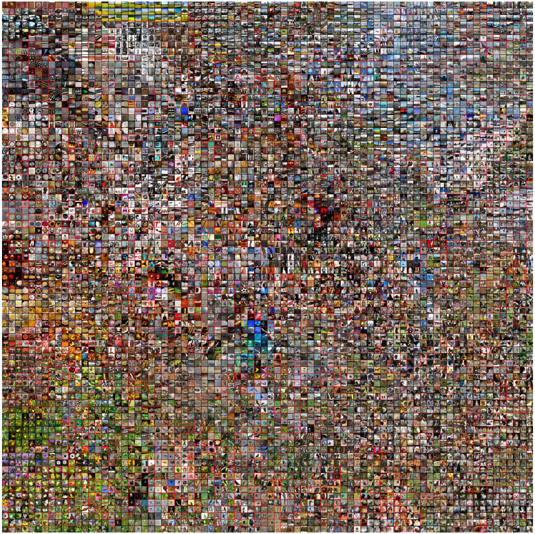



- AlexNet gives 4096 dimensional codes for each image
- t-SNE: place two codes close in 2D if they are close in 4096D

# t-SNE on ImageNet



http://cs.stanford.edu/people/karpathy/cnnembed/

## Visualizing Activations: DeConvolutional Approach

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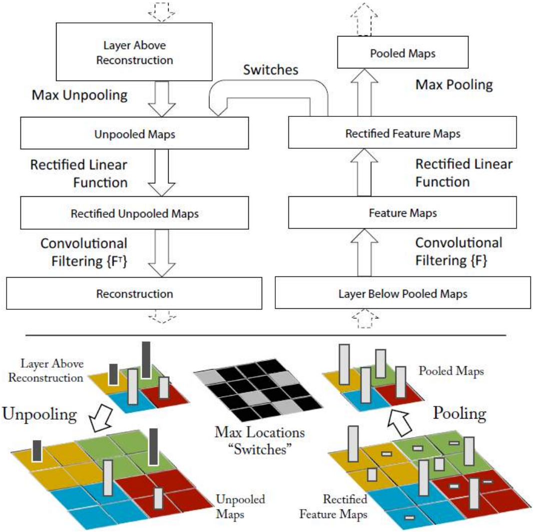
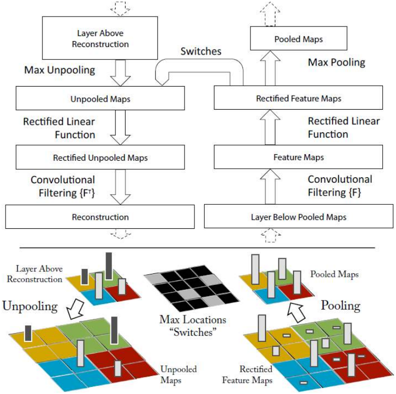
- A Deconvolutional Network is a convnet model run in reverse (runs all the same operations)
- PS: There are many later papers that improve the approach of Zeiler and Fergus (say using guided backprop), but the basic idea is similar

# Deconvolutional Network



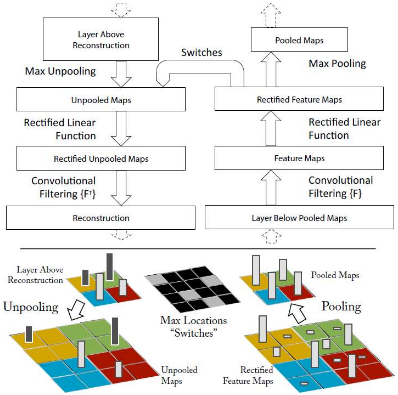
# Approach

- Attach a DeConv Net to a layer of the convnet (to be examined)
- Pass input image through CNN and obtain activations
- For a given neuron, set all activations to zero and backprop from there



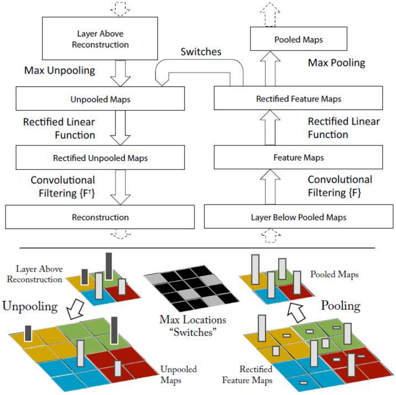
# Approach

- Successively unpool, rectify and filter till pixel space is reached
- Max is not invertible: Keep switch variables to keep track of locations of max in each pooling



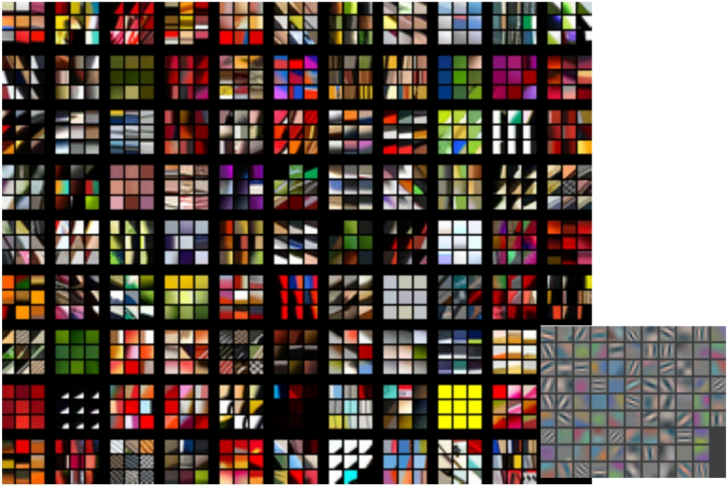
# Approach

- ReLU: Pass the signal through a ReLU non-linearity
- Filtering: Use learned filters to convolve backward signal, but use transposed versions of filters and applied to ReLU activations
- Reconstructions show which parts of input image are discriminative



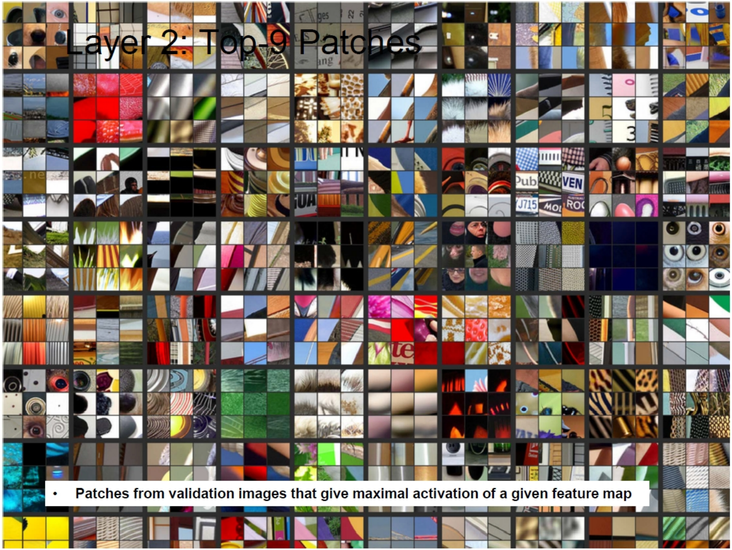
Feature Visualizations from last time were generated by this approach

# Layer 1 filters



Matthew Zeiler and Rob Fergus

# Layer 2 Patches



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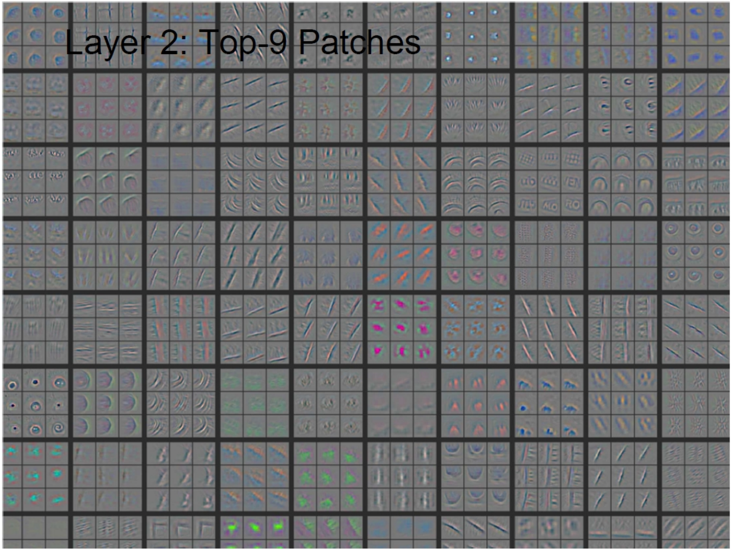
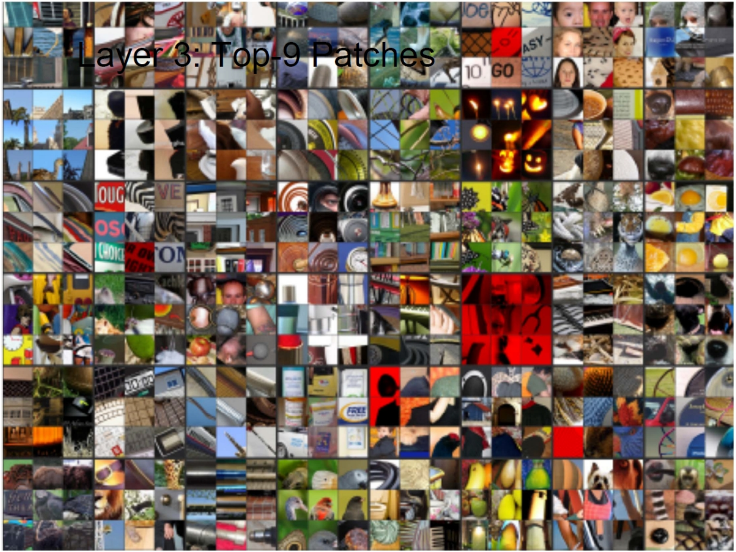


# Layer 3 Patches



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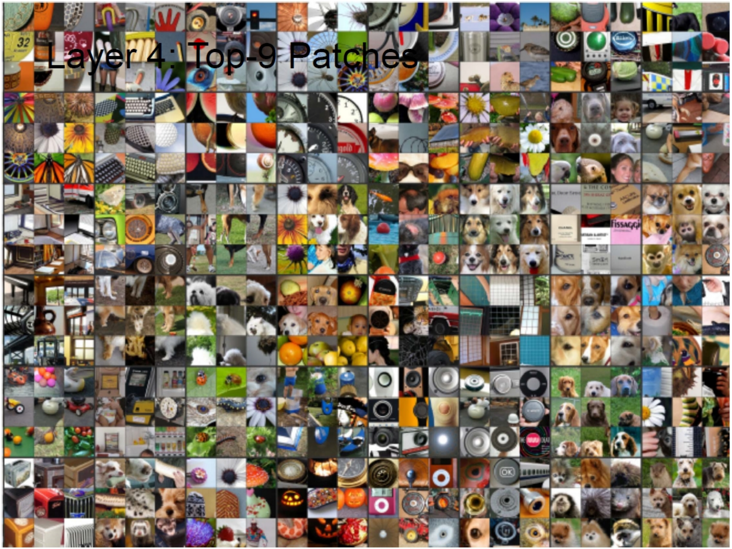

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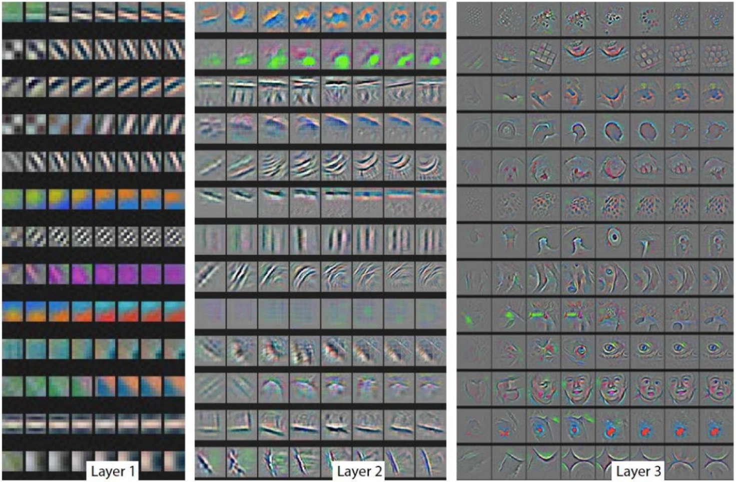
# Evolution of Filters



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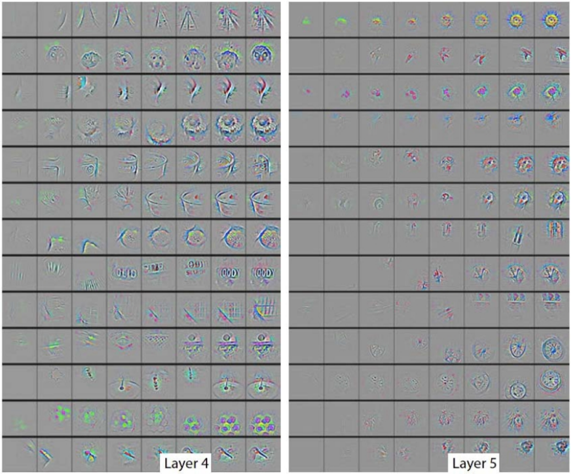


Caveat?

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- Zeiler and Fergus also used feature visualizations to see if network really identified the object or depended on context

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- Approach:Occlude images at different locations and visualize feature activations and classifier confidence

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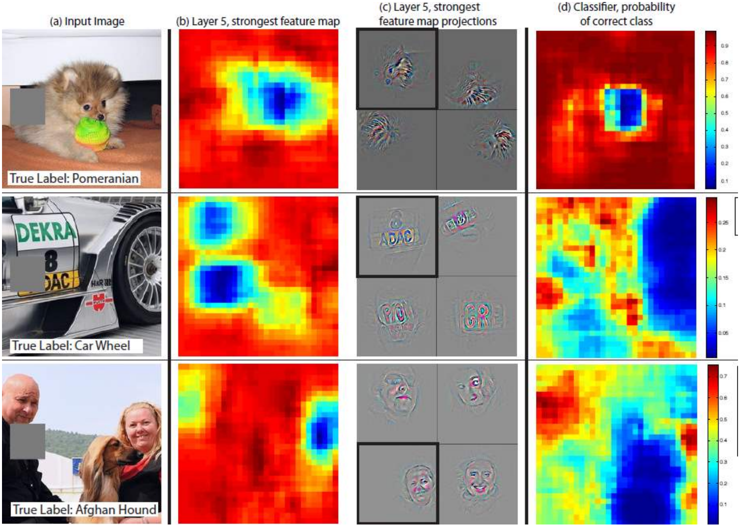

# Class Saliency Visualization

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- $\mathbf{w} = \left.\frac{\partial S_c}{\partial I}\right|_{I_0}$ is the gradient evaluated at $I_0$
- Magnitude of $w_i$ determines pixel influence (take max if multiple color channels)

# Visualizing Gradients

- If we occlude pixels denoted by black pixels in original image, we won't mess up the network's prediction

# Visualizing Gradients

- If we occlude pixels that represent the class spatial support, we will!

Generating an Image

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- Problem: Find a $L2$ regularize image $I$:

$$\arg\max_I S_c(I) - \lambda \|I\|_2^2$$

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- Question: How do we find an *image*?

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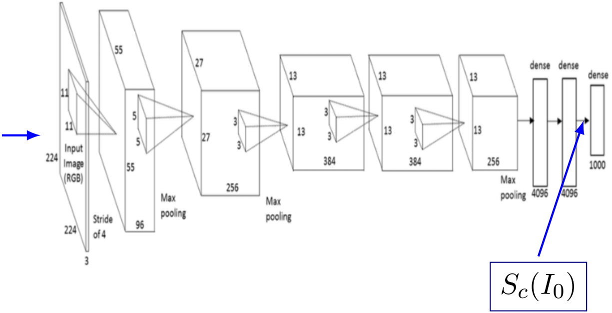
- Just do backpropagation!
  - Earlier: Used backpropagation to update weights
  - Now: Fix the network, pass image through network, obtain $S_c(I)$, go back and update pixels
- What should be the initial image?

# Class Model Visualization

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |




1. Fix network, input image $I_0$

# Class Model Visualization

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
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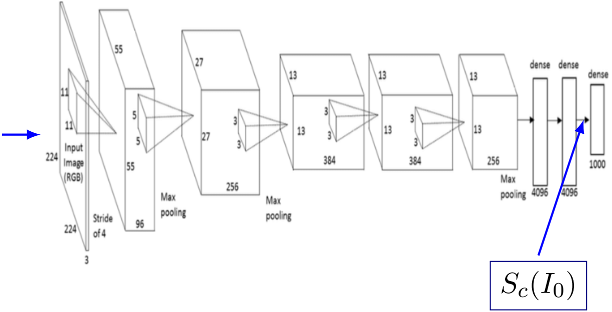


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2. Obtain $S_c(I_0)$ (Reminder: Unnormalized log probability for $c$)

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| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
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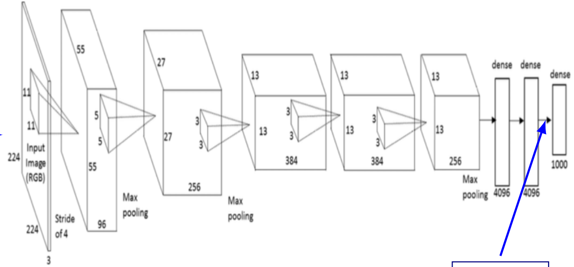


$S_c(I_0)$

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3. Update image by backpropagation

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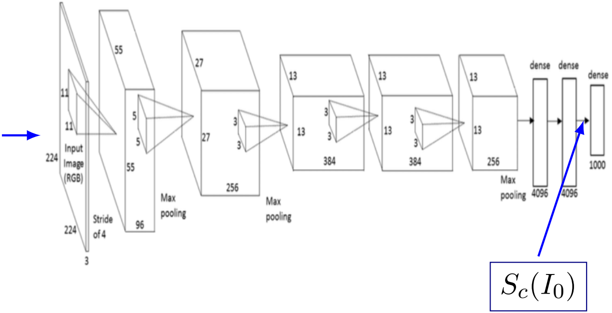
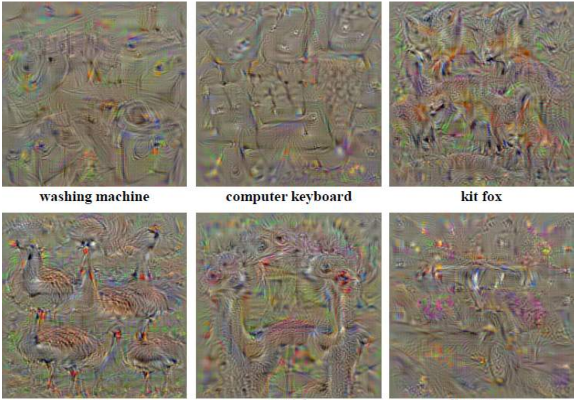



1. Fix network, input image $I_0$
2. Obtain $S_c(I_0)$ (Reminder: Unnormalized log probability for $c$)
3. Update image by backpropagation
4. Repeat till convergence

# Class Model Visualization:Examples



washing machine, computer keyboard, kit fox, goose, ostrich, limousine

# Changing the Regularizer

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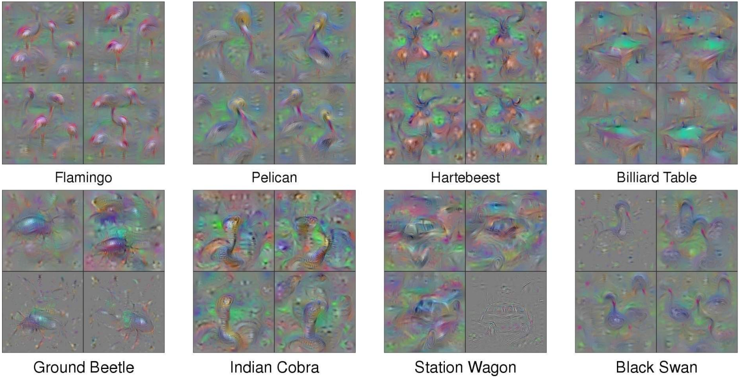
- Problem:Introduces high frequency artifacts
- Another regularizer: Blur $I$ at each update
- Also clip pixel values with small contributions

# Class Model Visualization with Blur



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- Process remains the same

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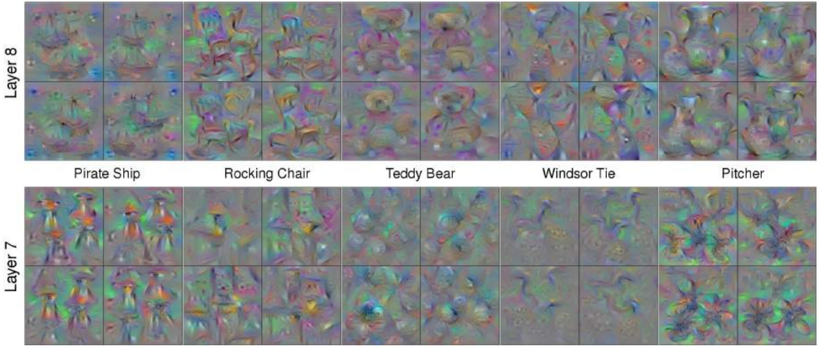
$$\arg\max_I a_i(I) - \lambda R(I)$$

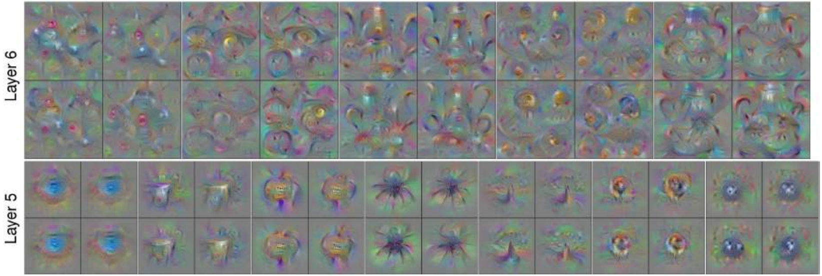
- Process remains the same
- Can use this to probe what each neuron likes!

# Example Features




- Layer 8 visualization is a class model (same as before)

# Example Features



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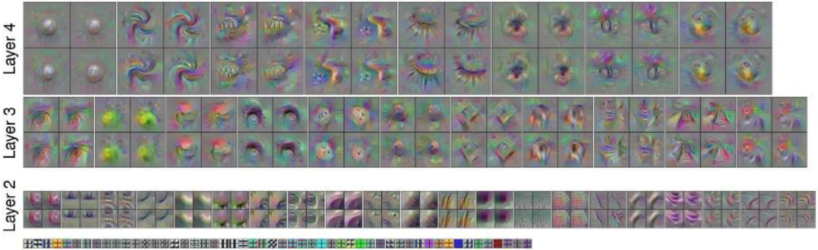


https://www.youtube.com/watch?v=AgkfIQ4IGaM

# A Short Digression

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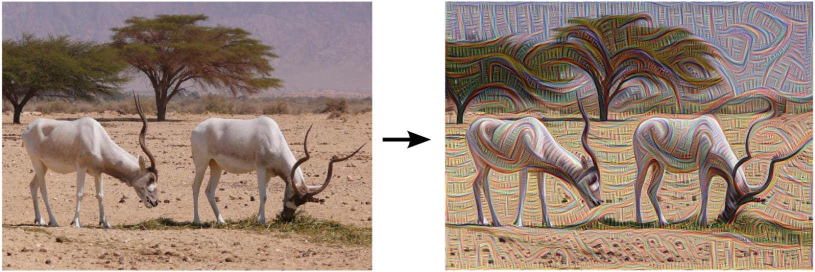
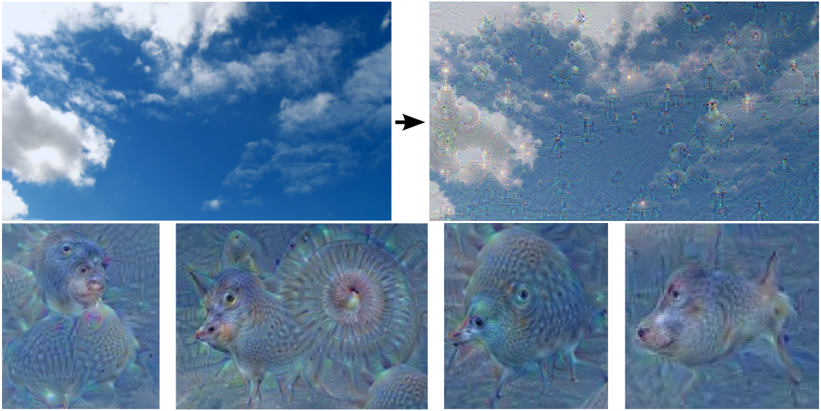
- What would happen?

# Deep Dream



Lower layers detect edges etc., on optimization such features will get boosted up

# Deep Dream



"Admiral Dog!" "The Pig-Snail" "The Camel-Bird" "The Dog-Fish"

Slightly higher layers start to overinterpret shapes in images

# Deep Dream


Slightly higher layers start to overinterpret shapes in images

# Iterating

The input for these were noise images!

Video: Grocery store trip:
https://www.youtube.com/watch?v=DgPaCWJL7XI

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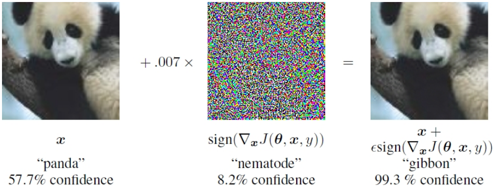
- Adversarial Examples! ($\Delta \mathbf{x}$ is an image)



# Convolutions: Motivation

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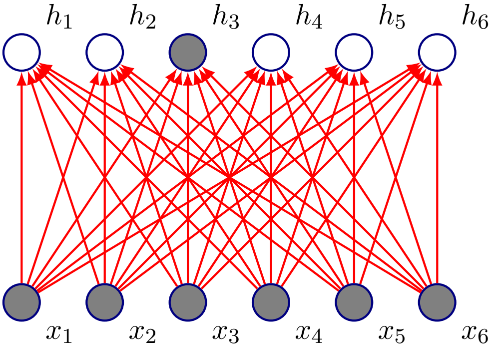
  - Convolutional networks have *sparse interactions* by making kernel smaller than input
  - $\implies$ need to store fewer parameters, computing output needs fewer operations ($O(m \times n)$ versus $O(k \times n)$)

# Motivation: Sparse Connectivity




- Fully connected network: $h_3$ is computed by full matrix multiplication with no sparse connectivity

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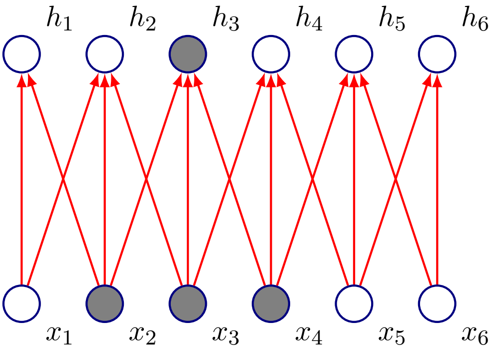


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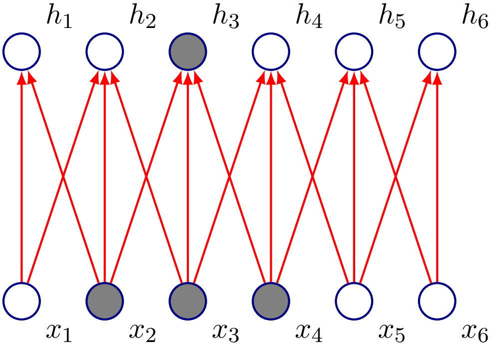


- Kernel of size 3, moved with stride of 1
- $h_3$ only depends on $x_2, x_3, x_4$

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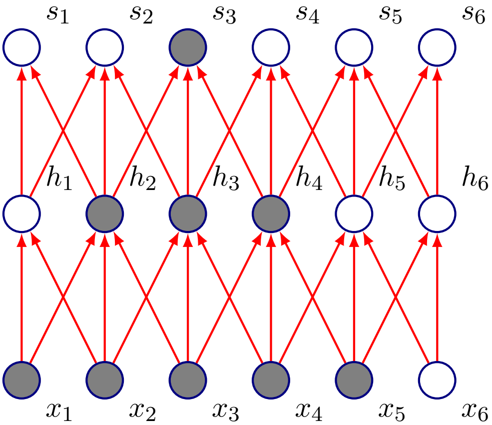


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- Storage improves dramatically as $k \ll m, n$

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- Equivariance: $f$ is equivariant to $g$ if $f(g(\mathbf{x})) = g(f(\mathbf{x}))$
- The form of parameter sharing used by CNNs causes each layer to be equivariant to translation
- That is, if $g$ is any function that translates the input, the convolution function is equivariant to $g$

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- This property is useful when we know some local function is useful everywhere (e.g. edge detectors)
- Convolution is not equivariant to other operations such as change in scale or rotation

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- If input is translated by small amount: values of most pooled outputs don't change

# Pooling: Invariance

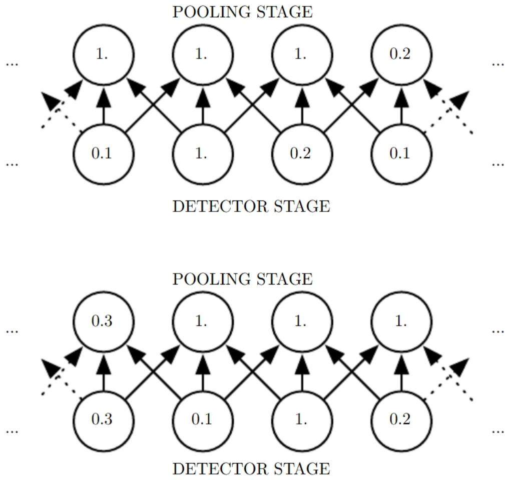


Figure: Goodfellow *et al.*

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- Pooling over spatial regions produces invariance to translation, what if we pool over separately parameterized convolutions?
- Features can learn which transformations to become invariant to (Example: Maxout Networks, Goodfellow *et al* 2013)
- One more advantage: Since pooling is used for downsampling, it can be used to handle inputs of varying sizes

# Variations

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- Unshared Convolution: Kernel is different at every location
- No parameter sharing!

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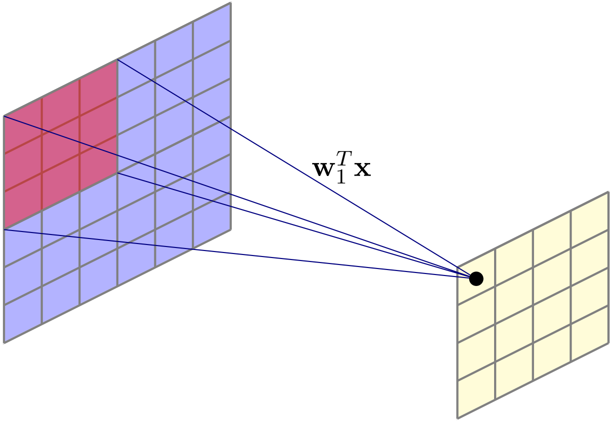

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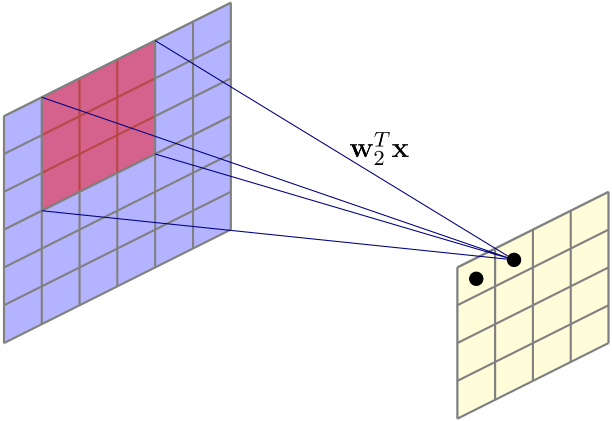

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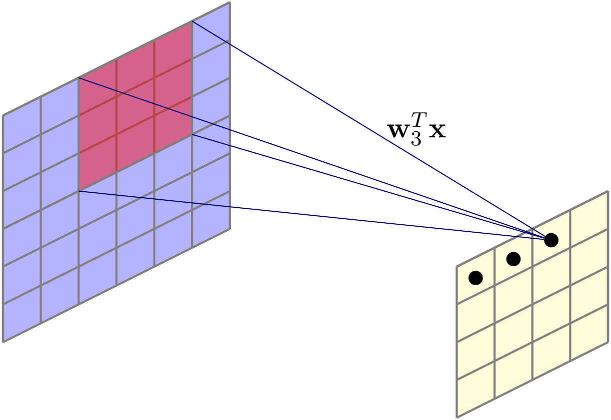

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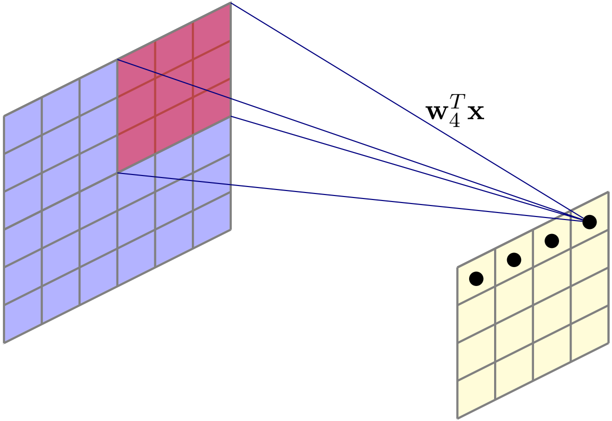

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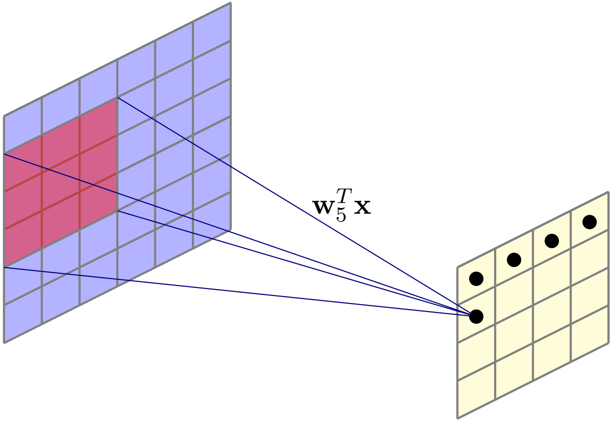

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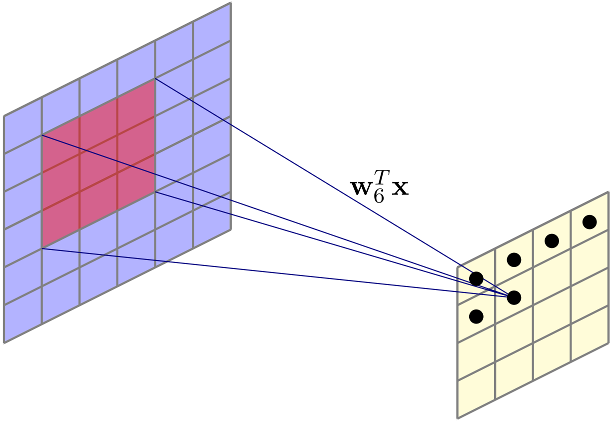

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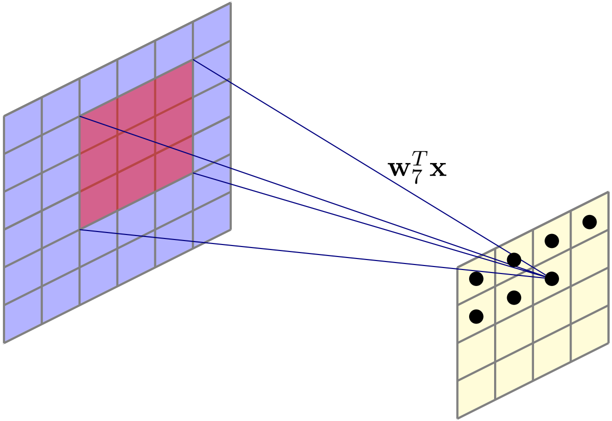

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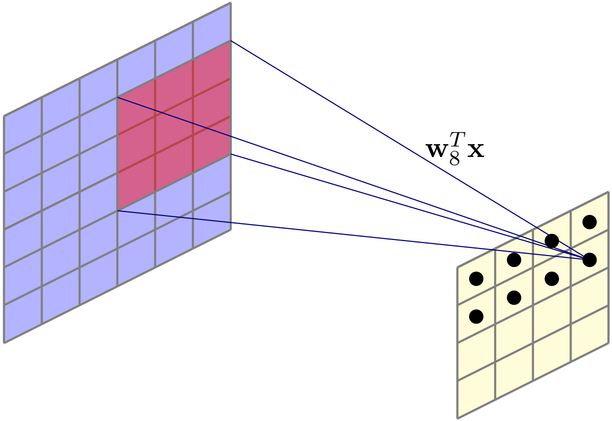

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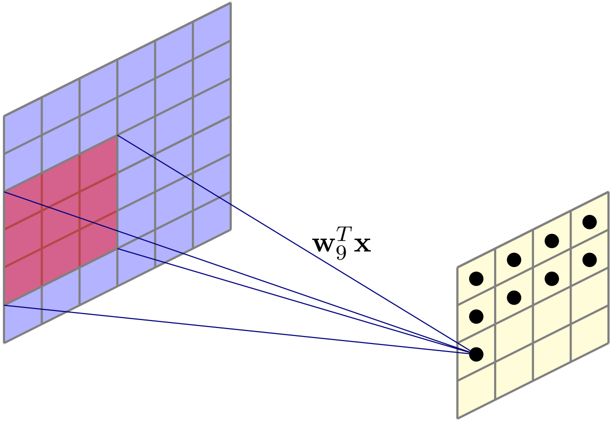

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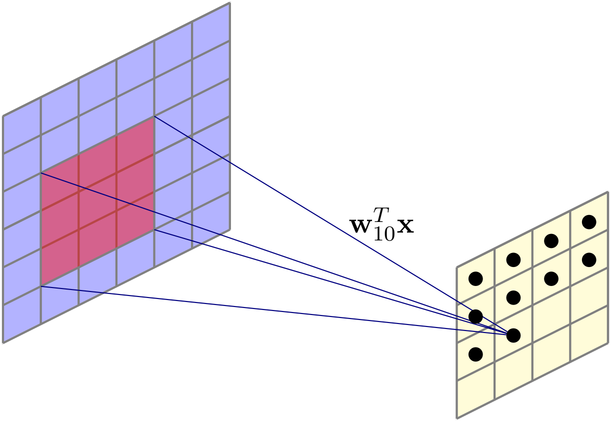

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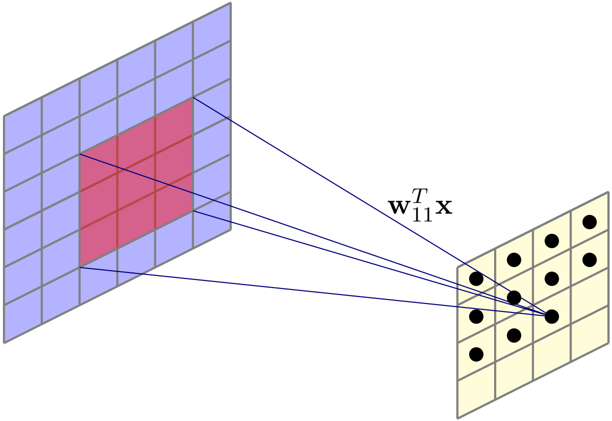

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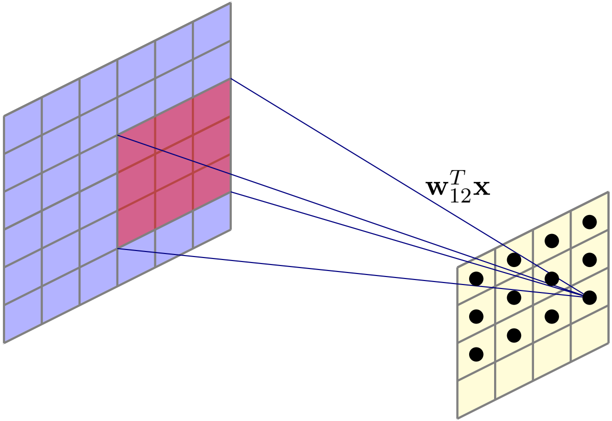

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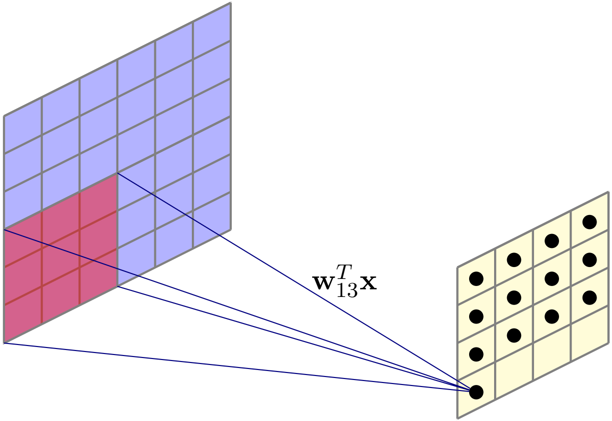

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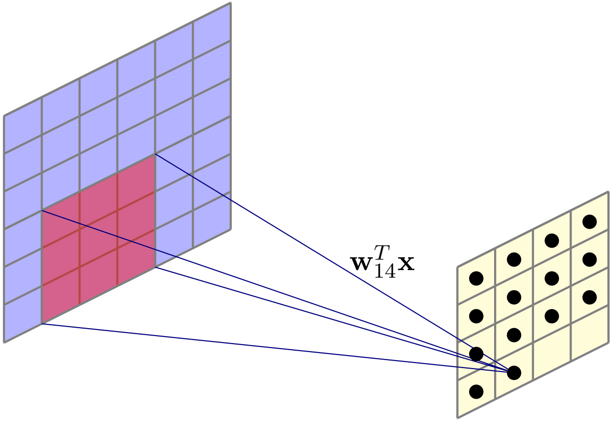

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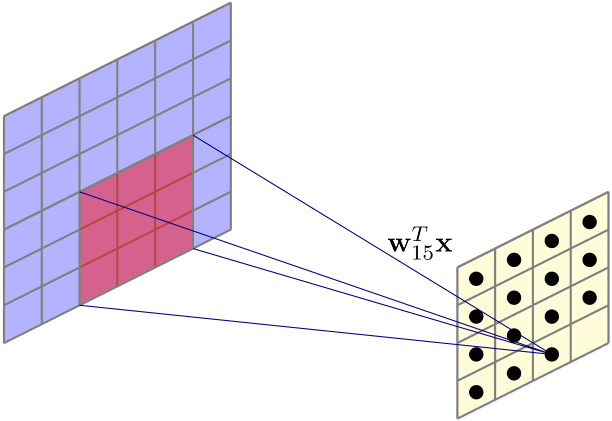

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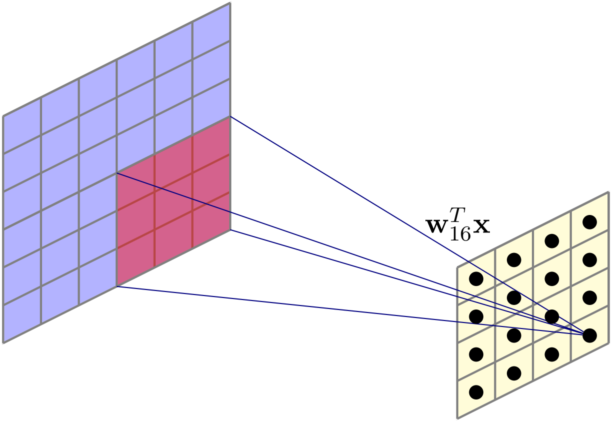


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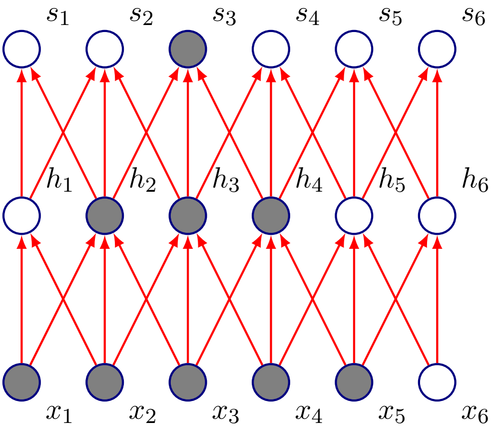
- Idea: Have a set of kernels and rotate them while traversal
- Ensures that immediate neighbors have different kernels
- Some parameters sharing (for 5 kernels in the previous example, what is the number of parameters?)

Dilated Convolutions

*à trous*: Convolutions with Holes

# A Problem with regular convolutions




- Connections in CNNs are sparse, but units in deeper layers are connected to more of the input. At what rate does the effective receptive field size increase with depth?

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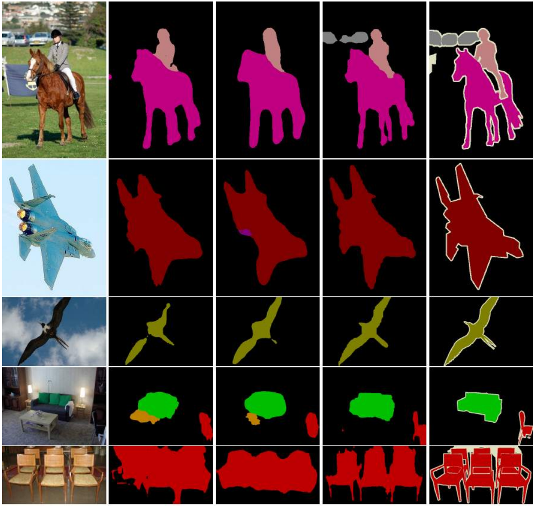
- Some examples?

# Semantic Segmentation



(a) Image (b) FCN-8s (c) DeepLab (d) Our front end (e) Ground truth

Image: *Multiscale Context Aggregation by Dilated Convolutions*, Yu and Koltun, ICLR 2016

# WaveNet: Causal Convolutions

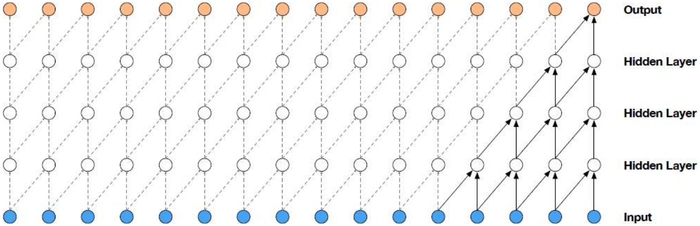



Image: *WaveNet: A Generative Model for Raw Audio*, Oord *et al.*, ICLR 2016

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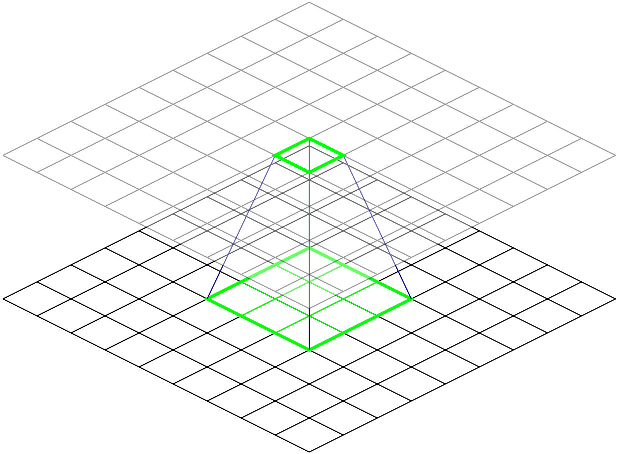
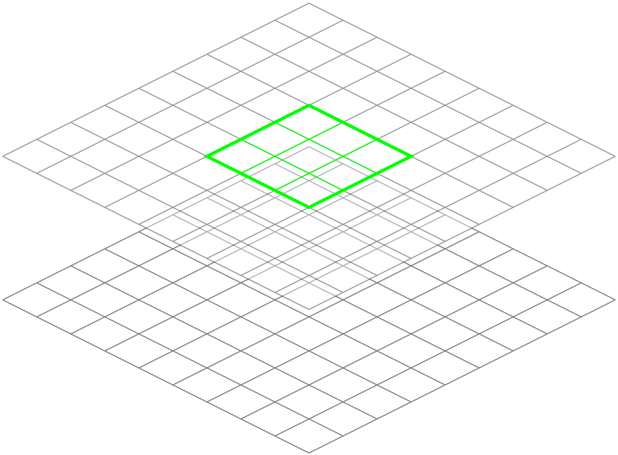
- Very old idea going to the 80s wavelet theory literature

# Regular Convolution

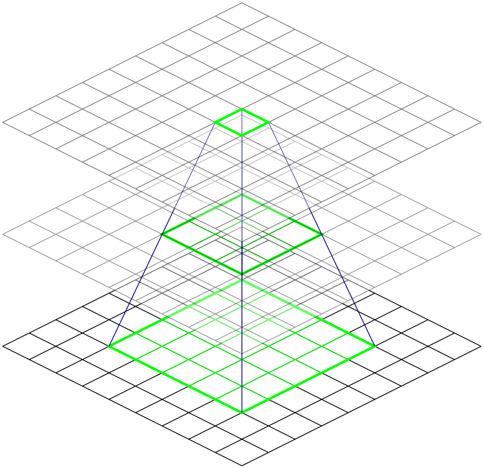


- The unit on the second layer has a receptive field of size $3 \times 3$

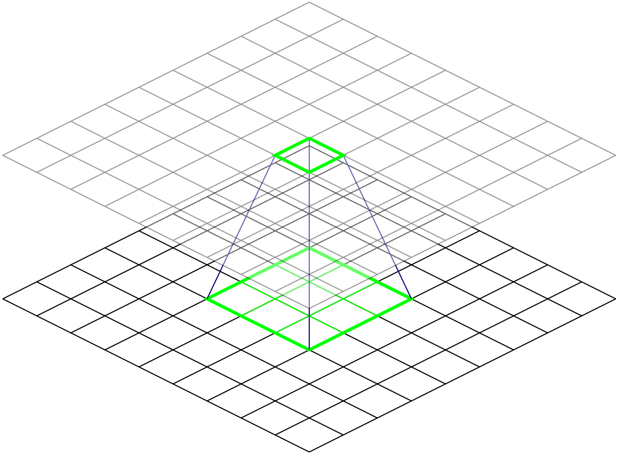
# Regular Convolution



# Regular Convolution

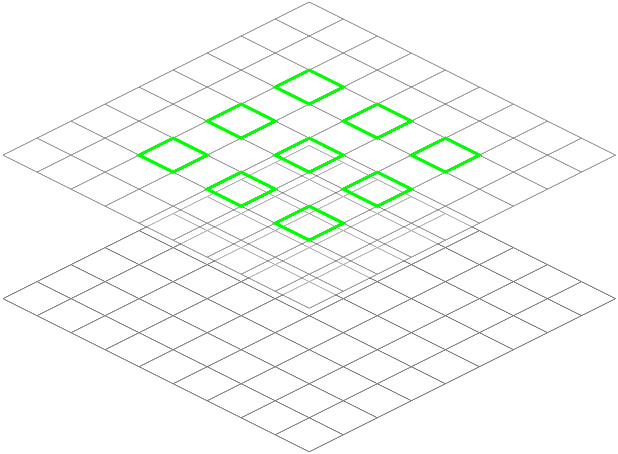


- The unit on the third layer has an effective receptive field of size $5 \times 5$

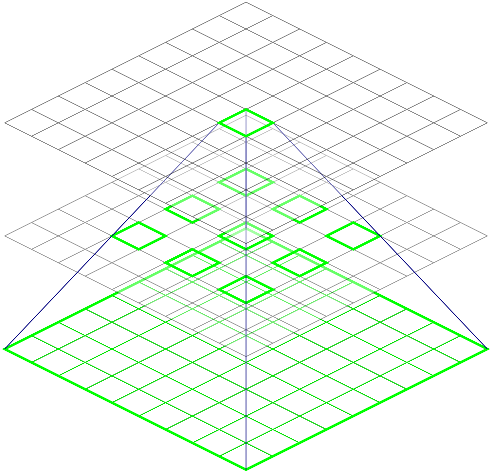


- The unit on the second layer has a receptive field of size $3 \times 3$

# Dilated Convolution: Dilation of 1



# Dilated Convolution



- The unit on the second layer has a receptive field of size $9 \times 9$

# WaveNet: Dilated Causal Convolutions

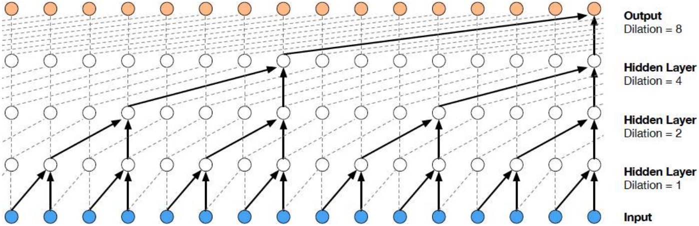


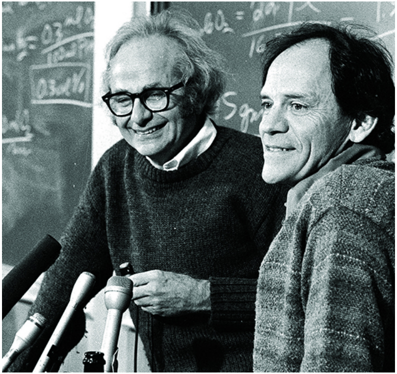

Image: *WaveNet: A Generative Model for Raw Audio*, Oord *et al.*, ICLR 2016

- We will see this in detail a few classes later

## The Neuroscientific Motivation for Convolutional Networks

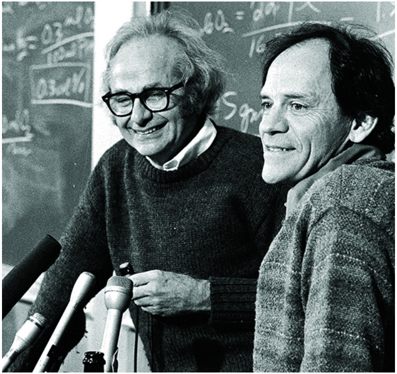
## Idea Genealogy for CNNs

# Hubel-Wiesel Experiments, 1959



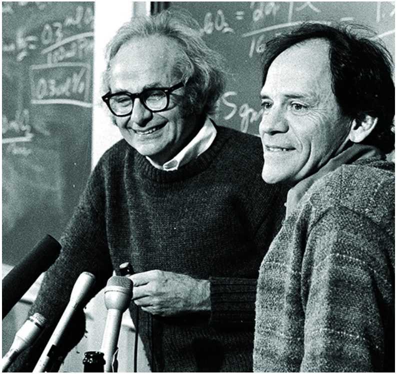
- David Hubel and Torsten Wiesel did a set of famous experiments to determine basic facts about mammalian vision

# Hubel-Wiesel Experiments, 1959



- David Hubel and Torsten Wiesel did a set of famous experiments to determine basic facts about mammalian vision
- Example: Recorded activity of individual neurons and observed responses to images projected in precise locations on a screen in front of the cat

# Hubel-Wiesel Experiments, 1959



- Neurons in the cat's early visual system responded very strongly to specific patterns of light, such as oriented bars and almost not at all to other patterns

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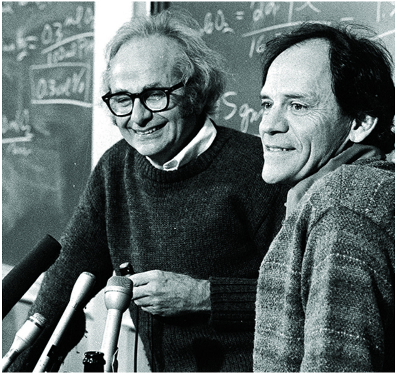


- Neurons in the cat's early visual system responded very strongly to specific patterns of light, such as oriented bars and almost not at all to other patterns
- Neurons in the later visual system responded to more complex stimuli and responses also exhibited invariance to translations etc

# A Simplified View of Brain Function

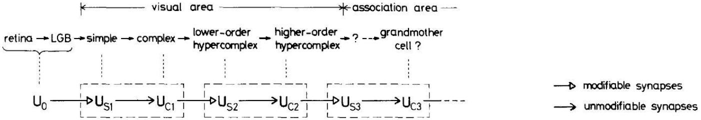
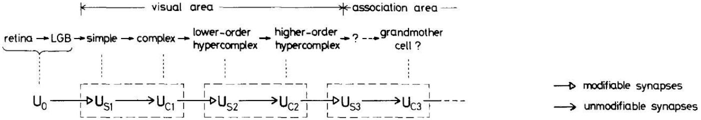



Image Source: Fukushima, 1980

- Images are projected onto the retina, neurons in retina do some simple preprocessing but do not substantially alter the representation

# A Simplified View of Brain Function

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- The signal channels into the area LGN (through the optic nerve)

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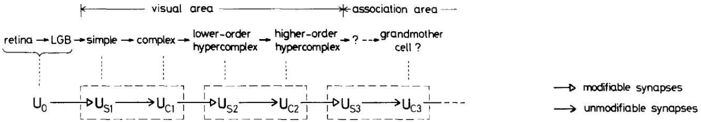



Image Source: Fukushima, 1980

- Images are projected onto the retina, neurons in retina do some simple preprocessing but do not substantially alter the representation
- The signal channels into the area LGN (through the optic nerve)
- Let's assume these regions simply carry the signal from eye to area V1

# A Simplified View of Brain Function

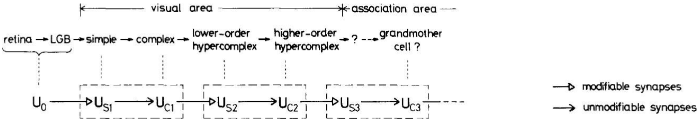



Image Source: Fukushima, 1980

- **V1 is arranged in a spatial map:**
  - 2D structure that mirrors structure of image in the retina

# A Simplified View of Brain Function

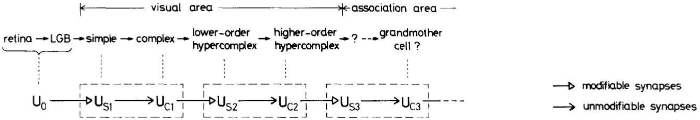



Image Source: Fukushima, 1980

- **V1 is arranged in a spatial map:**
  - 2D structure that mirrors structure of image in the retina
  - Light incident in the lower half of the retina only affects the lower half of V1

# A Simplified View of Brain Function

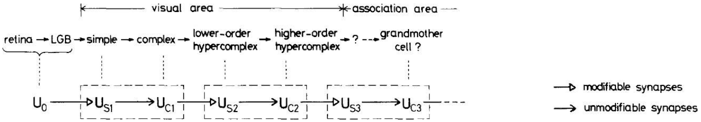
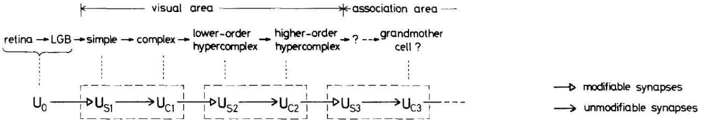



Image Source: Fukushima, 1980

- **V1 has many simple cells:** Roughly characterized by a linear function of image in small, spatially localized receptive fields (detection)

# A Simplified View of Brain Function

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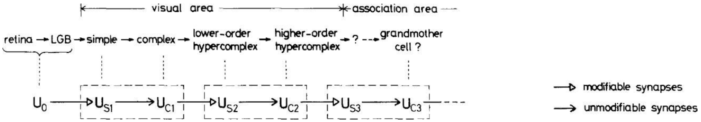


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- **V1 has many simple cells:** Roughly characterized by a linear function of image in small, spatially localized receptive fields (detection)
- **V1 has many complex cells:** Features detected similar to simple cells, but invariant to small shifts in position of feature (pooling)
- Also invariant to some changes in lighting

# A Simplified View of Brain Function

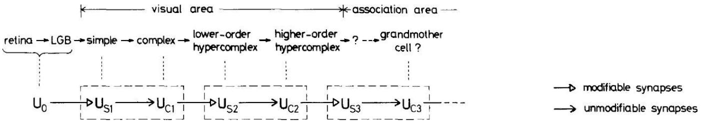



Image Source: Fukushima, 1980

- In the simplified view, this basic strategy is repeated many times

# A Simplified View of Brain Function

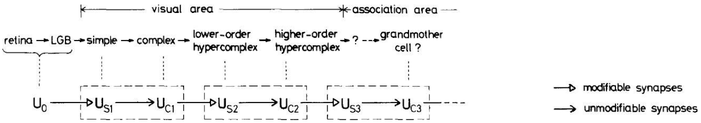



Image Source: Fukushima, 1980

- In the simplified view, this basic strategy is repeated many times
- After multiple layers, we find cells that respond to only specific concepts and are invariant to many transformations of the input (grandmother cells in the medial temporal lobe)

# Simple and Complex Cells

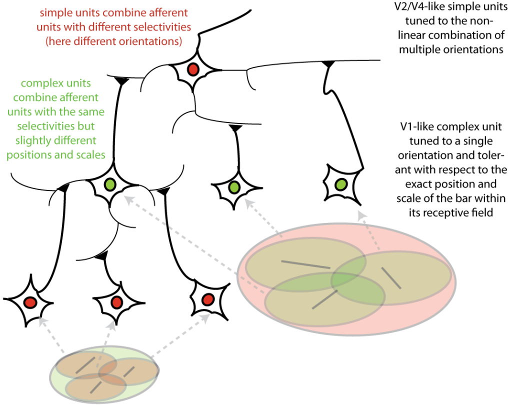
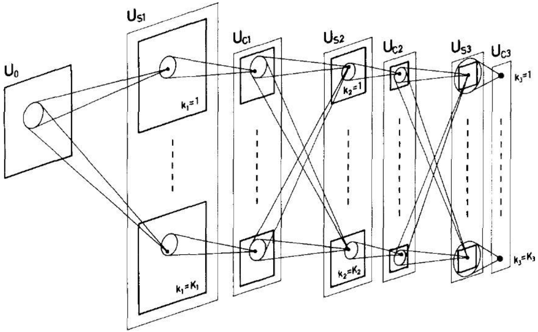


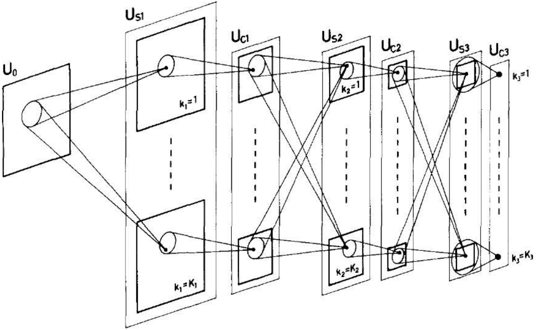

Image Source: Scholarpedia

# Neocognitron (Fukushima, 1980)




- Fukushima used this simplified view of brain function to build a neural network

# Neocognitron (Fukushima, 1980)




- Fukushima used this simplified view of brain function to build a neural network
- Was trained by an unsupervised procedure

# TDNNs and CNNs

- Waibel and Hinton introduced a 1-D Convolutional Network and trained it by backpropagation

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- Waibel and Hinton introduced a 1-D Convolutional Network and trained it by backpropagation
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# TDNNs and CNNs

- Waibel and Hinton introduced a 1-D Convolutional Network and trained it by backpropagation
- Convolutional Networks topology was directly inspired by the Neocognitron which was directly inspired by the Hubel-Weisel model
- TDNNs inspired the use of backpropagation for training for 2D CNNs (Yann LeCun, 1989)

# Next time

- More on Equivariance
- Group Equivariant CNNs
- Spatial Transformers and related ideas
- Back to Architectures: Ultra Deep Models
- Begin: CNNs on Graphs and Combinatorial Data

Quiz!