Question 0.1. You are given a rooted tree $T$. The root contains a message $m$ which has to be communicated to all nodes in $T$. Communicating $m$ from $u$ to $v$ ($uv \in T$) takes one timestep. Furthermore each node can only communicate with one of its neighbours in a given timestep. Note that in one timestep many nodes may transfer $m$ to a corresponding neighbour in parallel. Design an algorithm to determine the the minimum number of timesteps $C(T)$ to distribute the message throughout $T$.

This is problem 16 on pg. 327 of the Kleinberg & Tardos book.
Here is a sketch of the solution.

Suppose $r$ is the root of $T$. Further suppose that we know the optimal way to schedule the communications for each subtree $T_i$ for $i \in N(r)$ (the neighbours of $r$). A natural strategy is to simply communicate $m$ to each $i \in N(r)$ from $r$ in decreasing order of communication time $C(T_i)$ (ties broken arbitrarily). This suggests a natural linear time bottom-up algorithm.

To see that the strategy is optimal we simply use induction on the depth $d$ of $T$. The base case $d = 0$ is trivial. Suppose the hypothesis holds for trees of depth atmost $d-1$. Now one can show (say by contradiction) that $T$ with depth $d$ has an optimal communication strategy given by the algorithm above.