let \( a : R \) in which \( R \) denotes the type of the field of all the real numbers, an abstract structure of a vector space \( V \) can be defined by a quadruple \(<V, F, \cdot, +>\), where \( V \) denotes a set, \( F \) denotes a field, \( \cdot \) denotes a type operator that \( \cdot : F \times V \to V \), and \( + \) denotes another type operator that \( + : V \times V \to V \). Please note that we will use symbols like \( x, y, z \) etc. for the elements in \( V \), and \( a, b, c \) etc. for those in \( F \).

For example, let \( x_1, ..., x_d : V \), and suppose \( x_1, ..., x_d \) are linearly independent, we have \( \forall y : V, \ y = a_1 \cdot x_1 + ... + a_d \cdot x_d \).

One most often used set of \( V \) is \( R^d \) (read as "d fold cartesian of \( R \)"). Due to the existence of zero element in \( F \), a vector space always contains a zero vector. We can intuitively define the concept of orthogonality: two vectors \( a_1 \) and \( a_2 \) are orthogonal in the case when \( a_1 \cdot a_2 = 0 \), i.e. the inner product of \( a_1 \) and \( a_2 \) is zero. For example, considering \( a_1, a_2 : R^2 \), let \( a_1 = (\theta_1, \theta_2) \) and \( a_1 = (\tau_1, \tau_2) \), we have \( \theta_1 \cdot \tau_1 + \theta_2 \cdot \tau_2 = 0 \).

Note: In type theory, there is no well-formed mathematical representation for inner product so far.

Linear Transformation \( x \to \rho(x) \): A linear transformation between two vector spaces \( V \) and \( W \) is a map \( \rho : V \to W \) such that the following hold:
1. \( \rho(v_1 + v_2) = \rho(v_1) + \rho(v_2) \) for any vectors \( v_1 \) and \( v_2 \) in \( V \), and
2. \( \rho(\alpha v) = \alpha \rho(v) \) for any scalar \( \alpha \). For example:
Let’s define $\mathfrak{F}(V)$, which yields inner product on $V :\langle V, F, \cdot, + \rangle$. Suppose $V$ is isomorphic to $U$ (i.e., if $x + y = z$ in $V$, then $\rho(x) + \rho(y) = \rho(z)$ in $U$), we have that $\mathfrak{F}(V)$ is isomorphic to $\mathfrak{F}(U)$.

Claim: any two $d$-dimensional vector spaces are isomorphic, i.e. $R^d$.

There is no natural basis for vector space, i.e. all basis are equivalent.

In type theory, vector space is presented as

$$\exists \alpha \left( \cdot : \beta \times \alpha \rightarrow \alpha \right)$$

(1)