CMSC 39600-1 Online Algorithms Autumn 2004 Teacher: Adam Kalai

Homework due 10/8. Please write the name of any collaborator you worked with. You may not simply lookup the solution.

1. Consider a list of length n. Show that the competitive ratio of the Frequency Count algorithm (which keeps the list in sorted order according to the number of lookups) is $\Omega(n)$. In other words, give an example of a sequence of lookup requests such that the cost of Frequency Count is at least cn times larger than the cost of the best offline algorithm for handling the requests, for some constant c.

2. (from Avrim Blum)

Imagine you are standing next to a long fence, extending as far as you can see in both directions. You want to cross the fence and you know that somewhere it has a hole you can go through. But, you don't know whether the hole is to your right or left, or how far away it is. You want a strategy for finding the hole that does well using a competitive ratio measure.

Let's model this problem as follows: you are initially located at the origin on the real line. The hole is at some (positive or negative) integer coordinate n. You can move left or right at cost equal to the distance moved, and the game continues until you reach n. You then look at the ratio of your cost to $|n|$ (n is the optimal off-line cost since it is the distance you would have travelled if you had known where the hole was).

Here is a deterministic strategy with competitive ratio of 9 (which is optimal for deterministic strategies): move to location 1, then to -2, then to 4, then to -8, etc. The worst case is if $h = 2^{2i} + 1$ for some integer i. In this case, the online cost is $(1+1)+(2+2)+(4+4)+\ldots+(2^{2i+1}+2^{2i+1})+|n| < 9|n|$.

- Describe and analyze a randomized algorithm whose competitive ratio is at most 7. That is, for any n, the expected cost of the algorithm is at most $7n$.
- (harder) Describe and analyze a randomized algorithm whose competitive ratio is at most $7 - \epsilon$ for some constant $\epsilon > 0$. (Optimum is 4.511)

3. Imagine flipping a fair coin T times. Show that the expected number of ties (times when the number of heads equals the number of tails) is $O(\sqrt{T})$.