
CMCS 312: Programming Languages

Homework 5: Type Safety of PCF

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Due : November 9, 2006

Consider the following language.

$$\begin{aligned} \tau & ::= \text{unit} \mid \text{nat} \mid \tau + \tau \mid \tau \rightarrow \tau \\ n & ::= 0 \mid 1 \mid \dots \\ v & ::= \star \mid \text{inl}_{\tau+\tau} v \mid \text{inr}_{\tau+\tau} v \mid \text{fix } f(x : \tau) : \tau \text{ is } t \text{ end} \\ t & ::= v \mid x \mid t + t \mid t * t \mid \text{isZero } t \mid t t \mid \text{inl}_{\tau+\tau} t \mid \text{inr}_{\tau+\tau} t \mid \\ & \quad \text{case } t \text{ of } \text{inl}_{\tau+\tau} x \Rightarrow t \mid \text{inr}_{\tau+\tau} x \Rightarrow t \end{aligned}$$

This language is and its variants are commonly referred as PCF (Partial Computable Functions).

1. Give a type system.
2. Give a small-step semantics.
3. Give an encoding of booleans in PCF. For this you need to a) give a type τ_{bool} to represent the `bool` type, b) and to define a function `encode` that maps the values `true` and `false` to values of PCF, and maps the terms of the form `if t_1 then t_2 else t_3` to a term of PCF. Note that `encode` must be recursive because the subterms themselves can contain boolean terms.
4. Prove that the language is safe with respect to your type system and the operational semantics. You need to show the Inversion Lemma, the Canonical Forms Lemma, the Progress Theorem, the Permutation Lemma, the Weakening Lemma, the Substitution Lemma, and the Preservation Theorem.