Today

Robustifying
Augment with PCPP

First we'll robustify the LDT.

Robust-LDT: $A : \mathbb{F}^m \rightarrow \mathbb{F}$

1. Choose random plane $p$
   and accept if $A(p)$ is a deg d poly.

Claim: $S(A, P, d) > \delta \Rightarrow \text{Exp}[\|A(p)\|_{\infty}]$

Expected fractional distance of $A$ hom
satisfying the rob-LDT $\geq \delta - \epsilon_0$

$\mathbb{E}[S(A, P, d)] \geq \delta - \epsilon_0.$

Pf: Create planes oracle $A : \mathbb{F}^m \rightarrow \mathbb{P}_d$

$A(p) = \arg\max_{p \in \mathbb{P}_d} S(A(p), p)$

Thus, $S(A, P, d) = S(A, A(p))$

On the other hand,

$\delta - \epsilon_0 \leq \text{Pr}_{p}[\text{rej}] = \text{Pr}_{p}[A(p)(x) \neq A(x)] = \mathbb{E}_{p}[S(A, p, d)]$

Similarly, handled robust LDT.
PCP Verifier

Input: circuit C

Oracles: A: \(F^m \rightarrow F\)

p: \(F^{3m^3} \rightarrow F^{3m^3}\)

1. \(p' \in \mathbb{F}_2^{|m|}\) Check if \(A_\mathbf{p'} \in \mathbb{P}_{2d}\)

2. \(p \in \mathbb{F}_2^{3m^3}\) Check if \(\forall i: p_i^6 \in \mathbb{P}_{2d}\)

3. \(p \in \mathbb{P}_2\) \(\forall z = (a, y, u, o, a, b, c) \in \mathbb{P}\)

Check if

\[ \sum_{i \neq j} (A_{\mathbf{p}'} - a)(A_{\mathbf{p}'} - a) = \sum_{i \neq j} (z_i - z_j)(z_i - z_j) \]

As before:

Randomness = \(O(\log n)\)

Query Complexity = \(poly \log n\)

Soundness Analysis

Claim: For \(\epsilon\)

If \(A\) is \(\epsilon\)-far from any degree \(m\) poly \(A: F^m \rightarrow F\)

if \(A'_{\mathbf{p}'}\) is a bad assign, then expected fraction of input seed that needs to be modified = \(\Omega(\epsilon - \epsilon)\).

\(\Rightarrow\) \(\leq \) prob \(\Omega(\epsilon - \epsilon)\) at least \(\Omega(\epsilon - \epsilon)\) portion

\(\Rightarrow\) seed needs to be modified.
Augmenting with a POP $\delta$ proximally.

Case (i) $S(A, P_{\text{nd}}) > \varepsilon$

In exp. e.e. of the up read by Step 1 needs to be changed.

i.e., In exp. e.e. of total up

Case (ii) $S(P, P_{\text{OC(3m+3)}}) > \varepsilon$

Same as above.

Case (iii) $A \in P_{\text{nd}} \implies S(A, a) \leq \varepsilon$

$> P_{\text{nd}} \implies S(P_{\text{nd}}, \ldots, P_{\text{POP}}, \ldots, P_{\text{9m}}) \leq \varepsilon$.

3 bad set where identity does not hold.

$$\frac{181}{111^m} > 1 - \frac{d - 4\varepsilon}{9}$$

All of $IB_{\text{np}}$ needs to be changed.

One need: $E[IB_{\text{np}}] > (1 - \frac{d - 4\varepsilon}{9})^m$.

Either $A_p$ or $P_p$ or all $g$ $B_{\text{np}}$ needs to be changed.

$$\left(\frac{111 - 4\varepsilon}{9}\right)^m > \frac{\varepsilon - \varepsilon}{5}$$

Thus, in expectation $\frac{\varepsilon - \varepsilon}{5}$ fraction of up reads needs to be changed.

Hence, $\varepsilon_{up} > \varepsilon - \varepsilon_0$ at least $\varepsilon - \varepsilon_0$ fraction of input needs to be changed.
Augmenting with a PCP of proximity

\[ \text{a: } F^m \rightarrow F \text{ contains the assignment to sp} \]

Need to cross verify against \( W: \mathbb{Z}^n \rightarrow \{0,1\} \)

Rob-PCPP Verifier

1. Run Rob-PCPP Verifier

2. Proximity Test

Choose \( i \in [k] \) \( x \in H^m \) that corresponds \( x_i \)

Choose a random plane \( P \) thru \( i \)

Accept if \( \forall P \) is a low degree poly \( P \) \( A(x) = W(i) \)

Soundness Analysis

Case (i) \( A \) is not \( 2\epsilon \)-close to any \( \alpha \in \mathcal{P}_{md} \) s.t. \( \langle \alpha | \mathcal{H}_n \rangle = 1 \)

Then \( \text{Pr}_{P \leftarrow \text{Rob-PCPP-sets}}[\epsilon > \frac{\delta}{10}] = \Omega(1), \) at least \( \epsilon \sim \Omega(1) \) of proof oracle read needs to be modified.
Case (ii) $A$ is $2\varepsilon_o$-close to a st. $C(a_{th_o}) = 1$

then $W$ is S-flat from $a_{th_o}$

Hence $a I_{prob}$