**SUMMARY**
- We propose and analyze a convex optimization-based algorithm for recovering a low-rank tensor (multi-way array) from noisy measurements.
- Our result shows that while the bound for the previously analyzed overlapped Schatten 1-norm depends on the average (square root) of the mode-ranks, the bound for the new latent Schatten 1-norm depends on the minimal mode-rank.
- Thus it performs almost as well as knowing the mode with the lowest rank.
- The bound agrees well with simulation.

**OVERLAPPED SCHATTEN NORMS**
- Overlapped \( S_p \)/1-norm

\[
\|W - W^*\|_{S_p} = \sum_{i=1}^K \|W_{i:} - W_{i:}^*\|_{S_p}.
\]

- Studied by Gandy+11; Liu+09; Signoretto+10; T+10 for low-rank tensor recovery.

**LATENT SCHATTEN NORMS**
- Latent \( S_p \)/1-norm

\[
\|W - W^*\|_{S_p} = \sum_{i=1}^K \|\hat{W}_{i:} - \hat{W}_{i:}^*\|_{S_p}.
\]

**PROPERTY**
- The norm is bounded by the average (square root) of mode-ranks.

**BOUND FOR THE OVERLAPPED \( S_p/1 \)-NORM (\( \ast \)+11)**

Assume \( W \) has rank \( (r_1, \ldots, r_p) \). Given observation,

\[
Y = W + E,
\]

\([E \sim \text{Gaussian } N(0, \sigma^2 I)]\) noise) any minimizer \( \hat{W} \) of the convex problem

\[
\min_{W} \quad \|Y - W\|_F + \alpha \|W\|_{S_p}^p
\]

satisfies the bound

\[
\|W - W^*\|_F \leq c \left( \sum_{i=1}^p \|\hat{w}_{i:} - W_{i:}\|_{S_p} \right) + \alpha \frac{\|W^*\|_{S_p}^p}{\|W\|_{S_p}^p}
\]

with high probability if

\[
\alpha = \frac{c}{\left( \sum_{i=1}^p \|\hat{w}_{i:} - W_{i:}\|_{S_p} \right)}
\]

**PROPERTY**
- More general: Latent \( S_p/q \)-norm

**BOUND FOR THE LATENT \( S_p/1 \)-NORM (NEW)**

Assume \( \text{TV} \) has rank \( (r_1, \ldots, r_p) \). Given observation,

\[
Y = W + E,
\]

satisfies the bound

\[
\|W - W^*\|_F \leq c \left( \sum_{i=1}^p \|\hat{w}_{i:} - W_{i:}\|_{S_p} \right) + \alpha \frac{\|W^*\|_{S_p}^p}{\|W\|_{S_p}^p}
\]

with high probability if

\[
\alpha = \frac{c}{\left( \sum_{i=1}^p \|\hat{w}_{i:} - W_{i:}\|_{S_p} \right)}
\]

**DUALITY**

**EXPERIMENT**
- Goal: recover a synthetic rank \((r_1, r_2)\) tensor of size \([50 \times 50 \times 20]\).
- Error measured by \(\|Y - \hat{Y}\|_F\).

**NUMERICAL COMPARISON**
- Randomly generated problems with various ranks and sizes.
- Tucker rank complexity = (1); Latent rank complexity = (2).
- Latent Schatten 1-norm performs better in most cases.