Problemset 3
TTIC 31100 / CMSC 39000 Computational geometry

November 8, 2010

Problem 1. Recall that $\ell_1^d$ denotes the space $\mathbb{R}^d$, endowed with the $\ell_1$ norm. Describe an algorithm which given a set of $n$ points in $\ell_1^d$, computes their diameter in time $O(2^d \cdot n)$. Hint: Use an embedding into $\ell_\infty$ of appropriate dimension.

Problem 2. Show that any embedding of the $n$-cycle into the line, has distortion $\Omega(n)$.

Problem 3. Recall that $K_{3,3}$ is the complete bipartite graph with each side having 3 vertices. Let $G$ be the graph obtained from $K_{3,3}$ after replacing every edge with a path of length $n$. Show that any embedding of the shortest-path metric of $G$ into the Euclidean plane, has distortion $\Omega(n)$.

Problem 4. Let $(X, d)$ be the uniform metric on $n$ points. I.e. $X = \{x_1, \ldots, x_n\}$, and for any $i \neq j \in \{1, \ldots, n\}$, $d(x_i, x_j) = 1$. Show that for any fixed $d \geq 1$, any embedding of $(X, d)$ into $\mathbb{R}^d$, has distortion $\Omega(n^{1/d})$. Hint: It might be helpful to prove the assertion first for the case $d = 1$. 