Homework 3

1 Properties of Rademacher Averages

Given a function class $\mathcal{F}$, define,

$$c\mathcal{F} := \{cf \mid f \in \mathcal{F}\} \ ,$$

$$\mathcal{F} + g := \{f + g \mid f \in \mathcal{F}\} \ ,$$

where $c \in \mathbb{R}$, $g : \mathcal{X} \to \mathbb{R}$. What are $\mathfrak{R}_n(c\mathcal{F})$ and $\mathfrak{R}_n(\mathcal{F} + g)$ in terms of $\mathfrak{R}_n(\mathcal{F})$?

2 Margin Bound

Let $\mathcal{F}$ be a class of real-valued functions $\{(X_i, Y_i)\}_{i=1}^n$ be a data set. Let $\gamma > 0$ and denote the number of "margin mistakes" of $f \in \mathcal{F}$ by,

$$M_\gamma(f) := |\{i \mid Y_i f(X_i) < \gamma\}| \ .$$

Prove that, with probability at least $1 - \delta$, for all $f \in \mathcal{F}$,

$$L(f) \le \frac{M_\gamma(f)}{n} + \frac{2}{\gamma}\mathfrak{R}_n(\mathcal{F}) + 2\sqrt{\frac{\log(1/\delta)}{2n}} \ ,$$

where $L(f) := \mathbb{P}\left(Y f(X) < 0\right)$.

Hint: Consider the function $\phi(t)$ that is 1 when $t < 0$, 0 when $t > \gamma$ and $1 - t/\gamma$ when $t \in [0, \gamma]$.

3 Pollard's Pseudodimension

For $\mathcal{F} \subseteq [0,1]^{\mathcal{X}}$, recall the definition of $\mathrm{Pdim}(\mathcal{F})$ from Lecture 15. Show that an equivalent definition is

$$\mathrm{Pdim}(\mathcal{F}) := \mathrm{VCdim}\left(\{(x, r) \mapsto \mathrm{sgn}(f(x) - r) \mid f \in \mathcal{F},\ r \in [0,1]\}\right) \ .$$