Homework Assignment 2

TTIC 31010 and CMSC 37000-1

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Problem 1.
(a) Suppose that we are given two arrays of bits, $A = (a_0, \ldots, a_{n-1})$ and $B = (b_0, \ldots, b_{k-1})$, where $k \leq n$. Let $c_j = \sum_{i=0}^{k-1} a_{j+i} b_i$ for $j \in \{0, \ldots, n-k\}$. Give an algorithm that computes $C = (c_0, \ldots, c_{n-k})$ in time $O(n \log n)$.
(b) Suppose that we are given a string of bits $A = (a_0, \ldots, a_{n-1})$ and a "pattern" $B = (b_0, \ldots, b_{k-1})$ (where $k \leq n$). Every b_i is either 0, 1 or a special character $\star$ ("a wildcard character"). We say that B matches A at position $j \in \{0, \ldots, n-k\}$ if for every $i \in \{0, \ldots, k-1\}$ either $b_i = a_{i+j}$ or $b_i = \star$. For example, the string "0011011" matches the pattern "$0 \star 1$" at positions 0, 1 and 4.

Give an algorithm that outputs the list of all positions j at which B matches A in time $O(n \log n)$.

Hint: Let us say that b_i is a mismatched character of type 0 at position j if $b_i = 0$ and $a_{j+i} = 1$; b_i is a mismatched character of type 1 at position j if $b_i = 1$ and $a_{j+i} = 0$. Let c_j^0 and c_j^1 be the number of mismatched characters of type 0 and 1, respectively, at position j. Show how to find the values of all c_j^0 and c_j^1 (for $j \in \{0, \ldots, n-k\}$) in time $O(n \log n)$.

Problem 2. Let $G = (V, E)$ be an arbitrary directed graph, with a source s, a sink t, and a positive integer capacity $c(e)$ on every edge $e \in E$. Decide whether the following statement is true or false. If it is true, give a proof, and if it is false, show a counterexample.

- If f is a maximum s-t flow in G, then f saturates every edge in $out(s)$ with flow. That is, for each $e \in out(s)$, $f(e) = c(e)$.

Problem 3. Removed (see Problem 1 in Homework 3).