Homework Assignment 2
TTIC 31010 and CMSC 37000-1
January 24, 2012

Problem 1.
(a) Suppose that we are given two arrays of bits, \( A = (a_0, \ldots, a_{n-1}) \) and \( B = (b_0, \ldots, b_{k-1}) \), where \( k \leq n \). Let \( c_j = \sum_{i=0}^{k-1} a_{j+i} b_i \) for \( j \in \{0, \ldots, n-k\} \). Give an algorithm that computes \( C = (c_0, \ldots, c_{n-k}) \) in time \( O(n \log n) \).

(b) Suppose that we are given a string of bits \( A = (a_0, \ldots, a_{n-1}) \) and a “pattern” \( B = (b_0, \ldots, b_{k-1}) \) (where \( k \leq n \)). Every \( b_i \) is either 0, 1 or a special character \( \star \) (“a wildcard character”). We say that \( B \) matches \( A \) at position \( j \in \{0, \ldots, n-k\} \) if for every \( i \in \{0, \ldots, k-1\} \) either \( b_i = a_{j+i} \) or \( b_i = \star \). For example, the string “0011011” matches the pattern “0\( \star \)1” at positions 0, 1 and 4.

Give an algorithm that outputs the list of all positions \( j \) at which \( B \) matches \( A \) in time \( O(n \log n) \).

Hint: Let us say that \( b_i \) is a mismatched character of type 0 at position \( j \) if \( b_i = 0 \) and \( a_{j+i} = 1 \); \( b_i \) is a mismatched character of type 1 at position \( j \) if \( b_i = 1 \) and \( a_{j+i} = 0 \). Let \( c_j^0 \) and \( c_j^1 \) be the number of mismatched characters of type 0 and 1, respectively, at position \( j \).
Show how to find the values of all \( c_j^0 \) and \( c_j^1 \) (for \( j \in \{0, \ldots, n-k\} \)) in time \( O(n \log n) \).

Problem 2. Let \( G = (V, E) \) be an arbitrary directed graph, with a source \( s \), a sink \( t \), and a positive integer capacity \( c(e) \) on every edge \( e \in E \). Decide whether the following statement is true or false. If it is true, give a proof, and if it is false, show a counterexample.

- If \( f \) is a maximum \( s-t \) flow in \( G \), then \( f \) saturates every edge in \( out(s) \) with flow. That is, for each \( e \in out(s) \), \( f(e) = c(e) \).

Problem 3. Removed (see Problem 1 in Homework 3).