You can discuss homework problems with other students, but you must write solutions on your own. This homework is due on Monday, February 6.

**Definition.** The $\ell_\infty$ distance between two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ is equal to $\|u-v\| = \max(|u_1 - v_1|, |u_2 - v_2|)$. The $\ell_\infty$ distance between two sets of points $U \subset \mathbb{R}^2$ and $V \subset \mathbb{R}^2$ is

$$d(U, V) = \inf_{u \in U, v \in V} \|u - v\|_\infty = \inf_{u \in U, v \in V} \max(|u_1 - v_1|, |u_2 - v_2|).$$

**Problem 1.** We are given a set of $n$ points in the plane. Design an algorithm that finds a pair of points with maximum $\ell_\infty$ distance in time $O(n)$. Prove the correctness of your algorithm.

**Problem 2.** Design an algorithm that given a set of $n$ axis–parallel rectangles in the plane finds a pair of rectangles with minimum $\ell_\infty$ distance in time $O(n \log n)$. Prove the correctness of your algorithm.

*Partial credit:* Solve the following simpler problem for a partial credit. Design an algorithm that given a set of $n$ axis–parallel rectangles in the plane and a parameter $t$ finds a pair of rectangles with $\ell_\infty$ distance less than $t$ in time $O(n \log n)$. If there is no such pair of rectangles, the algorithm should output that. Prove the correctness of your algorithm.

**Problem 3.** Design an algorithm for the following problem. The algorithm is given a set $C$ of $m$ circles and a set $P$ of $n$ points in $\mathbb{R}^2$. A circle in $C$ may lie within another circle in $C$, but no two circles may intersect. The algorithm must report all circles $C \in C$ that contain at least one point from $P$ in time $O((m + n) \ln(m + n))$. Prove the correctness of your algorithm.

![Figure 1](image-url)