

TTIC 106: Problem Set 2

Due Friday, February 5

1. A context free grammar in Chomsky normal form is a set of productions of the form $X \rightarrow YZ$ and $X \rightarrow a$ where X , Y , and Z are nonterminal symbols, and a is a terminal symbol. We write $X \rightarrow^* \alpha$ to mean that α is a string of terminal symbols that can be generated from the nonterminal X by repeatedly replacing a nonterminal Y by the right hand side of some production form Y .

We now also consider a directed graph where each edge of the graph is labeled with a terminal symbol. We will write $n \xrightarrow{a} m$ to indicate that there is an edge from node n to node m labeled with nonterminal a .

We assume a specified “start node” s .

Give inference rules for deriving assertions of the form $s \xrightarrow{X} m$ where s is the start node, X is a nonterminal symbol, and m is any other node and where this assertion states that there exists a path in the graph from s to m whose terminal labels form a string α with $X \rightarrow^* \alpha$.

State the running time of your rule set as a function of the number of nonterminals in the given grammar, the number of productions in the grammar, the number of nodes in the graph, and the number of edges in the graph. Try to make the rule set as efficient as possible.

2. Give a “Machine readable” definition of the field of real numbers as Dedekind cuts. This should include the definition of the type “cut” and a definition of zero, one, multiplication, and addition.

3. Give an axiomatic definition of the natural numbers $0, 1, 2, \dots$ as a structure $\{N : \mathbf{type}, \text{ zero} : N, \text{ successor} : N \rightarrow N\}$. Prove that your definition specifies the natural numbers up to isomorphism.