Lecture 1 — Introduction to Probability

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1 Basic Probability — Additional Exercises

Suppose, there is a very rare disease in the world which affects only 0.01% population of the world. There is a test which detects it up to accuracy of 1%, i.e if you have the disease than the test always outputs YES but even if you don't have it, the test still outputs YES with probability 0.01.

Exercise 1 (Bayes Rule) *What is the probability that you have the disease* given *that the test said* YES*?*

Question 1 *What additional assumptions did you make? Why do you think that the probability of you having the disease is* 0.01%*? What is the probability of you having the disease if your prior belief was that you have this disease with probability* 0*?*

Exercise 2 (Bayes Rule + Independent Events) *You conduct the above test twice, and both the times the output is* YES*. What is the probability of you having the disease?*

Exercise 3 *You conduct the above test twice, the first output is* YES *and the second output is* NO*. What is the probability of you having the disease? How many times you have to conduct this test to be reasonably sure that you have/don't have this disease?*

Question 2 *What is* reasonable *here? How many times do you need to do this test to correctly identify if you have/ don't have the disease with probability* 99.9%*? Watch this educational video: https://www.y2u.be/R13BD8qKeTg*

2 Random Variables — Additional Exercises

In some societies, unfortunately, male child is more *'desired'* than a female child. Consider the case of the country X, where the gender ratio is 940: that is, there are 940 female children per 1000 male children. Assume that the sex of a new born child is a uniform random variable: that is, the child is a boy with probability 50% and girl with probability 50%.

Exercise 4 *A social-scientist friend of yours tries to explain this phenomenon as follows: she hypothesis that every couple continue having children until they get one male child. What is the expected gender ratio under this hypothesis?*

Exercise 5 *She has another explanation as well: she hypothesis that every couple continue having children until either they get one male child, or get two girl children. What is the expected gender ratio under this hypothesis?*

The above two problems are toy versions of what is known as *gambler's ruin phenomenon*. If you are into gambling, you may find this interesting :-)

Suppose, we start with n coins, and make a sequence of bets. For each bet, we win one coin with probability p and lose a coin with probability $1 - p$. We have to quite when we go broke, that is, we lose all the n coins. You may use a computer simulation to estimate the following probabilities.

Exercise 6 *We start with* $n = 10$ *coins in a Roullete game. It is well-known that in Roullete,* $p = 18/38 = 9/19$. *What is the probability that we win* 10 *coins before going broke?*

Exercise 7 *We start with* $n = 1,000,000,000$ *coins in a Roullete game. What is the probability that we win* 10 *coins before going broke? What is the probability that we win* $1,000$ *coins before going broke?*

Question 3 *Suppose, you have* $n = 1,000$ *coins in a Roullete game. Can you find a strategy that allows you to win at least* 10 *coins with probability more than half?*

You can read more about this phenomenon and solution to these exercises here:
https://web.mit.edu/neboat/Public/6.042/randomwalks.pdf