Tutorial 1: Greedy Algorithms

TTIC 31010 and CMSC CMSC 37000-1

January 3, 2012

Definition 1. *Recall that a cut in a graph $G = (V, E)$ is a partition of V into two disjoint sets S and T. We denote the cut between S and T by (S, T). We say that an edge $e = (u, v) \in E$ is cut by (S, T) if one endpoint of e lies in S and the other lies in T (that is, if either $u \in S$ and $v \in T$ or $u \in T$ and $v \in S$). The size of the cut is the number of edges cut by the cut.*

Problem 1. *Design an algorithm that given a graph $G = (V, E)$ finds a cut (S, T) of size at least $|E|/2$ in G. (Note that there might be many cuts of size at least $|E|/2$; the algorithm needs to find just one of them.) What is the running time of your algorithm?*

Problem 2. *We are given a set of n jobs $\{1, \ldots, n\}$. Each job i has start time $s(i) \geq 0$ and processing time $t(i) > 0$. Only one job can run on one machine at a given time. Determine the minimum number of machines required to schedule all the jobs. Also find the optimal schedule (the map from jobs to machines).*

This problem is discussed in *Kleinberg and Tardos* on pp. 122–125.